

Lis User Manual

Version 1.2.75



The Scalable Software Infrastructure Project
<http://www.ssisc.org/>

July 5, 2012

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Contents

0	Changes from Version 1.1	1
1	Introduction	2
2	Installation	2
2.1	System Requirements	2
2.2	Extracting Archive	3
2.3	Installing on UNIX and Compatible Systems	4
2.3.1	Configuring Source Tree	4
2.3.2	Compiling	4
2.3.3	Installing	8
2.4	Installing on Windows Systems	8
2.5	Test Programs	9
2.5.1	test1	9
2.5.2	test2	9
2.5.3	test3	9
2.5.4	test4	9
2.5.5	test5	10
2.5.6	etest1	10
2.5.7	etest2	10
2.5.8	etest3	10
2.5.9	etest4	10
2.5.10	etest5	11
2.5.11	spmvttest1	11
2.5.12	spmvttest2	11
2.5.13	spmvttest3	11
2.5.14	spmvttest4	12
2.5.15	spmvttest5	12
2.6	Restrictions	13
3	Basic Operations	14
3.1	Initializing and Finalizing	15
3.2	Operating Vectors	15
3.3	Operating Matrices	18
3.4	Solving Linear Equations	24
3.5	Solving Eigenvalue Problems	27
3.6	Sample Programs	29
3.7	Compiling and Linking	31
3.8	Running	33
4	Quadruple Precision Operations	34
4.1	Using Quadruple Precision Operations	34
5	Matrix Storage Formats	36
5.1	Compressed Row Storage (CRS)	36
5.1.1	Creating Matrices (for Serial and Multithreaded Environments)	36
5.1.2	Creating Matrices (for MPI Environment)	37
5.1.3	Associating Arrays	37
5.2	Compressed Column Storage (CCS)	38
5.2.1	Creating Matrices (for Serial and Multithreaded Environments)	38
5.2.2	Creating Matrices (for MPI Environment)	39
5.2.3	Associating Arrays	39

5.3	Modified Compressed Sparse Row (MSR)	40
5.3.1	Creating Matrices (for Serial and Multithreaded Environments)	40
5.3.2	Creating Matrices (for MPI Environment)	41
5.3.3	Associating Arrays	41
5.4	Diagonal (DIA)	42
5.4.1	Creating Matrices (for Serial Environment)	42
5.4.2	Creating Matrices (for Multithreaded Environment)	43
5.4.3	Creating Matrices (for MPI Environment)	44
5.4.4	Associating Arrays	44
5.5	Ellpack-Itpack generalized diagonal (ELL)	45
5.5.1	Creating Matrices (for Serial and Multithreaded Environments)	45
5.5.2	Creating Matrices (for MPI Environment)	46
5.5.3	Associating Arrays	46
5.6	Jagged Diagonal (JDS)	47
5.6.1	Creating Matrices (for Serial Environment)	48
5.6.2	Creating Matrices (for Multithreaded Environment)	49
5.6.3	Creating Matrices (for MPI Environment)	50
5.6.4	Associating Arrays	50
5.7	Block Sparse Row (BSR)	51
5.7.1	Creating Matrices (for Serial and Multithreaded Environments)	51
5.7.2	Creating Matrices (for MPI Environment)	52
5.7.3	Associating Arrays	52
5.8	Block Sparse Column (BSC)	53
5.8.1	Creating Matrices (for Serial and Multithreaded Environments)	53
5.8.2	Creating Matrices (for MPI Environment)	54
5.8.3	Associating Arrays	54
5.9	Variable Block Row (VBR)	55
5.9.1	Creating Matrices (for Serial and Multithreaded Environments)	55
5.9.2	Creating Matrices (for MPI Environment)	56
5.9.3	Associating Arrays	57
5.10	Coordinate (COO)	58
5.10.1	Creating Matrices (for Serial and Multithreaded Environments)	58
5.10.2	Creating Matrices (for MPI Environment)	59
5.10.3	Associating Arrays	59
5.11	Dense (DNS)	60
5.11.1	Creating Matrices (for Serial and Multithreaded Environments)	60
5.11.2	Creating Matrices (for MPI Environment)	61
5.11.3	Associating Arrays	61
6	Functions	62
6.1	Operating Vector Elements	62
6.1.1	lis_vector_create	62
6.1.2	lis_vector_destroy	63
6.1.3	lis_vector_duplicate	63
6.1.4	lis_vector_set_size	64
6.1.5	lis_vector_get_size	64
6.1.6	lis_vector_get_range	65
6.1.7	lis_vector_set_value	65
6.1.8	lis_vector_get_value	66
6.1.9	lis_vector_set_values	66
6.1.10	lis_vector_get_values	67
6.1.11	lis_vector_scatter	68
6.1.12	lis_vector_gather	68

6.1.13	lis_vector_copy	69
6.1.14	lis_vector_set_all	69
6.2	Operating Matrix Elements	70
6.2.1	lis_matrix_create	70
6.2.2	lis_matrix_destroy	70
6.2.3	lis_matrix_duplicate	71
6.2.4	lis_matrix_malloc	71
6.2.5	lis_matrix_set_value	72
6.2.6	lis_matrix_assemble	72
6.2.7	lis_matrix_set_size	73
6.2.8	lis_matrix_get_size	73
6.2.9	lis_matrix_get_range	74
6.2.10	lis_matrix_set_type	75
6.2.11	lis_matrix_get_type	75
6.2.12	lis_matrix_set_blocksize	76
6.2.13	lis_matrix_convert	76
6.2.14	lis_matrix_copy	77
6.2.15	lis_matrix_get_diagonal	77
6.2.16	lis_matrix_set_crs	78
6.2.17	lis_matrix_set_ccs	78
6.2.18	lis_matrix_set_msr	79
6.2.19	lis_matrix_set_dia	79
6.2.20	lis_matrix_set_ell	80
6.2.21	lis_matrix_set_jds	80
6.2.22	lis_matrix_set_bsr	81
6.2.23	lis_matrix_set_bsc	81
6.2.24	lis_matrix_set_vbr	82
6.2.25	lis_matrix_set_coo	82
6.2.26	lis_matrix_set_dns	83
6.3	Operating Vectors and Matrices	84
6.3.1	lis_vector_scale	84
6.3.2	lis_vector_dot	84
6.3.3	lis_vector_nrm1	85
6.3.4	lis_vector_nrm2	85
6.3.5	lis_vector_nrmi	86
6.3.6	lis_vector_axpy	87
6.3.7	lis_vector_xpay	87
6.3.8	lis_vector_axpyz	88
6.3.9	lis_matrix_scaling	88
6.3.10	lis_matvec	89
6.3.11	lis_matvect	89
6.4	Solving Linear Equations	90
6.4.1	lis_solver_create	90
6.4.2	lis_solver_destroy	90
6.4.3	lis_solver_set_option	91
6.4.4	lis_solver_set_optionC	94
6.4.5	lis_solve	95
6.4.6	lis_solve_kernel	96
6.4.7	lis_solver_get_status	97
6.4.8	lis_solver_get_iters	97
6.4.9	lis_solver_get_itersex	98
6.4.10	lis_solver_get_time	98
6.4.11	lis_solver_get_timeex	99

6.4.12	lis_solver_get_residualnorm	99
6.4.13	lis_solver_get_rhistory	100
6.4.14	lis_solver_get_solver	101
6.4.15	lis_get_solvername	101
6.5	Solving Eigenvalue Problems	102
6.5.1	lis_esolver_create	102
6.5.2	lis_esolver_destroy	102
6.5.3	lis_esolver_set_option	103
6.5.4	lis_esolver_set_optionC	106
6.5.5	lis_solve	106
6.5.6	lis_esolver_get_status	107
6.5.7	lis_esolver_get_iters	107
6.5.8	lis_esolver_get_itersex	108
6.5.9	lis_esolver_get_time	108
6.5.10	lis_esolver_get_timeex	109
6.5.11	lis_esolver_get_residualnorm	109
6.5.12	lis_esolver_get_rhistory	110
6.5.13	lis_esolver_get_evalues	111
6.5.14	lis_esolver_get_evectors	111
6.5.15	lis_esolver_get_esolver	112
6.5.16	lis_get_esolvername	112
6.6	Operating External Files	113
6.6.1	lis_input	113
6.6.2	lis_input_vector	113
6.6.3	lis_input_matrix	114
6.6.4	lis_output	114
6.6.5	lis_output_vector	115
6.6.6	lis_output_matrix	115
6.7	Other Functions	116
6.7.1	lis_initialize	116
6.7.2	lis_finalize	116
6.7.3	lis_wtime	116
References		117
A File Formats		119
A.1	Extended Matrix Market Format	119
A.2	Harwell-Boeing Format	119
A.3	Extended Matrix Market Format for Vectors	121
A.4	PLAIN Format for Vectors	121

0 Changes from Version 1.1

1. Added support for eigensolvers.
2. Changed specifications of the following APIs:
 - (a) Changed names of `lis_output_residual_history()` and `lis_get_residual_history()` to `lis_solver_output_rhistory()` and `lis_solver_get_rhistory()`, respectively.
 - (b) Changed origin of Fortran subroutines `lis_vector_set_value()` and `lis_vector_get_value()` to 1.
 - (c) Changed origin of Fortran subroutine `lis_vector_set_size()` to 1.
 - (d) Changed name of precision flag `-precision` to `-f`.

1 Introduction

Lis, a Library of Iterative Solvers for linear systems, is a parallel numerical library for solving the linear equations

$$Ax = b$$

and the standard eigenvalue problems

$$Ax = \lambda x$$

with real sparse matrices using iterative methods. The solvers available in Lis are listed in Table 1 and 2, and the preconditioners are listed in Table 3. The matrix storage formats are listed in Table 4.

Table 1: Linear Solvers

CG	CR
BiCG	BiCR[2]
CGS	CRS[3]
BiCGSTAB	BiCRSTAB[3]
GPBiCG	GPBiCR[3]
BiCGSafe[1]	BiCRSafe[4]
BiCGSTAB(1)	TFQMR
Jacobi	Orthomin(m)
Gauss-Seidel	GMRES(m)
SOR	FGMRES(m)[5]
IDR(s)[13]	MINRES[14]

Table 2: Eigensolvers

Power Iteration
Inverse Iteration
Approximate Inverse Iteration
Rayleigh Quotient Iteration
Subspace Iteration
Lanczos Iteration
Conjugate Gradient[18, 19]
Conjugate Residual [20]

Table 3: Preconditioners

Jacobi
SSOR
ILU(k)
ILUT[6, 7]
Crout ILU[8, 7]
I+S[9]
SA-AMG[10]
Hybrid[11]
SAINV[12]
Additive Schwarz
User defined

Table 4: Matrix Storage Formats

Compressed Row Storage	(CRS)
Compressed Column Storage	(CCS)
Modified Compressed Sparse Row	(MSR)
Diagonal	(DIA)
Ellpack-Itpack generalized diagonal	(ELL)
Jagged Diagonal	(JDS)
Block Sparse Row	(BSR)
Block Sparse Column	(BSC)
Variable Block Row	(VBR)
Dense	(DNS)
Coordinate	(COO)

2 Installation

This section describes the instructions for installing and testing Lis. We assume Lis being installed on a Linux cluster.

2.1 System Requirements

Installation of Lis requires a C compiler. Fortran interface requires a compiler which supports FORTRAN 77. AMG preconditioner requires a compiler which supports Fortran 90. For parallel computing environments, OpenMP or MPI-1 is used. Lis has been tested on the environments shown in Table 5 (see also Table 7).

Table 5: Major Tested Platforms

C Compilers	OS
Intel C/C++ Compiler 7.0, 8.0, 9.1, 10.1, 11.1, Intel C++ Composer XE	Linux Windows
IBM XL C/C++ V7.0, 9.0	AIX Linux
Sun WorkShop 6, Sun ONE Studio 7, Sun Studio 11, 12	Solaris
PGI C++ 6.0, 7.1, 10.5	Linux
gcc 3.3, 4.3	Linux Mac OS X Windows
Microsoft Visual C++ 2008, 2010	Windows
Fortran Compilers (Optional)	OS
Intel Fortran Compiler 8.1, 9.1, 10.1, 11.1, Intel Fortran Composer XE	Linux Windows
IBM XL Fortran V9.1, 11.1	AIX Linux
Sun WorkShop 6, Sun ONE Studio 7, Sun Studio 11, 12	Solaris
PGI Fortran 6.0, 7.1, 10.5	Linux
g77 3.3 gfortran 4.3, 4.4 g95 0.91	Linux Mac OS X Windows

2.2 Extracting Archive

Enter the following command to extract the archive files, where (\$VERSION) represents the version:

```
>gunzip -c lis-($VERSION).tar.gz | tar xvf -
```

It creates a directory `lis-($VERSION)` along with its subfolders as shown in Figure 1.

```
lis-($VERSION)
+ config
| configuration files
+ include
| header files
+ src
| source files
+ test
  test programs
```

Figure 1: Files contained in `lis-($VERSION).tar.gz`

2.3 Installing on UNIX and Compatible Systems

2.3.1 Configuring Source Tree

Run the following script to configure the source tree:

- default: `>./configure`
- specifying the installation destination: `>./configure --prefix=<install-dir>`

Table 6 shows the options which can be specified for the configuration. Table 7 shows the computing environments which can be specified by `TARGET`.

Table 6: Major Configuration Options (see `./configure --help` for complete list)

<code>--enable-omp</code>	Use OpenMP
<code>--enable-mpi</code>	Use MPI
<code>--enable-fortran</code>	Use Fortran API
<code>--enable-saamg</code>	Use SA-AMG preconditioner
<code>--enable-quad</code>	Use quadruple precision operation
<code>--enable-gprof</code>	Use gprof
<code>--enable-shared</code>	Build shared libraries
<code>--prefix=<install-dir></code>	Name of the installation destination directory
<code>TARGET=<target></code>	Computing environments
<code>CC=<c_compiler></code>	C compiler
<code>CFLAGS=<c_flags></code>	Compilation options for C compilers
<code>FC=<fortran90_compiler></code>	Fortran 90 compiler
<code>FCFLAGS=<fc_flags></code>	Compilation options for the Fortran 90 compiler
<code>LDFLAGS=<ld_flags></code>	Link options

2.3.2 Compiling

In the directory `lis-($VERSION)`, run the following command to generate executable files:

```
>make
```

To ensure that the library has been successfully built, enter as follows in the directory `lis-($VERSION)`:

```
>make check
```

It runs a test script using the executable files created in the `lis-($VERSION)/test` directory, which reads the data of the coefficient matrix and the right hand side vector from the file `test/testmat.mtx` and writes the solution of the linear equation $Ax = b$ obtained with the BiCG method into `test/sol.txt`, and the residual history into `test/res.txt`. If all the elements of the solution are 1, then the result is correct. The result on SGI Altix 3700 is shown below.

Table 7: Examples of Targets (see `configure` for complete list)

<target>	Equivalent options
cray_xt3	<code>./configure CC=cc FC=ftn CFLAGS="-O3 -B -fastsse -tp k8-64" FCFLAGS="-O3 -fastsse -tp k8-64 -Mpreprocess" FCLDFLAGS="-Mnomain" ac_cv_sizeof_void_p=8 cross_compiling=yes --enable-mpi ax_f77_mangling="lower case, no underscore, extra underscore"</code>
fujitsu_pq	<code>./configure CC=fcc FC=frt ac_cv_sizeof_void_p=8 CFLAGS="-O3 -Kfast,ocl,preex" FFLAGS="-O3 -Kfast,ocl,preex -Cpp" FCFLAGS="-O3 -Kfast,ocl,preex -Cpp -Am" ax_f77_mangling="lower case, underscore, no extra underscore"</code>
hitachi	<code>./configure CC=cc FC=f90 FCLDFLAGS="-lf90s" ac_cv_sizeof_void_p=8 CFLAGS="-Os -noparallel" FCFLAGS="-Oss -noparallel" ax_f77_mangling="lower case, underscore, no extra underscore"</code>
ibm_bg1	<code>./configure CC=blrts_xlc FC=blrts_xlf90 CFLAGS="-O3 -qarch=440d -qtune=440 -qstrict -I/bg1/BlueLight/ppcfloor/bglsys/include" FFFLAGS="-O3 -qarch=440d -qtune=440 -qsuffix=cpp=F -qfixed=72 -w -I/bg1/BlueLight/ppcfloor/bglsys/include" FCFLAGS="-O3 -qarch=440d -qtune=440 -qsuffix=cpp=F90 -w -I/bg1/BlueLight/ppcfloor/bglsys/include" ac_cv_sizeof_void_p=4 cross_compiling=yes --enable-mpi ax_f77_mangling="lower case, no underscore, no extra underscore"</code>
nec_es	<code>./configure CC=esmpic++ FC=esmpif90 AR=esar RANLIB=true ac_cv_sizeof_void_p=8 ax_vector_machine=yes cross_compiling=yes --enable-mpi --enable-omp ax_f77_mangling="lower case, no underscore, extra underscore"</code>
nec_sx9_cross	<code>./configure CC=sxmpic++ FC=sxmpif90 AR=sxar RANLIB=true ac_cv_sizeof_void_p=8 ax_vector_machine=yes cross_compiling=yes ax_f77_mangling="lower case, no underscore, extra underscore"</code>

default

```
matrix size = 100 x 100 (460 nonzero entries)
initial vector x = 0
precision : double
solver    : BiCG 2
precon    : none
storage   : CRS
lis_solve : normal end

BiCG: number of iterations    = 15 (double = 15, quad = 0)
BiCG: elapsed time           = 5.178690e-03 sec.
BiCG: preconditioner         = 1.277685e-03 sec.
BiCG: matrix creation        = 1.254797e-03 sec.
BiCG: linear solver          = 3.901005e-03 sec.
BiCG: relative residual 2-norm = 6.327297e-15
```

--enable-omp

```
max number of threads = 32
number of threads = 2
matrix size = 100 x 100 (460 nonzero entries)
initial vector x = 0
precision : double
solver    : BiCG 2
precon    : none
storage   : CRS
lis_solve : normal end

BiCG: number of iterations    = 15 (double = 15, quad = 0)
BiCG: elapsed time           = 8.960009e-03 sec.
BiCG: preconditioner         = 2.297878e-03 sec.
BiCG: matrix creation        = 2.072096e-03 sec.
BiCG: linear solver          = 6.662130e-03 sec.
BiCG: relative residual 2-norm = 6.221213e-15
```

```
--enable-mpi --
number of processes = 2
matrix size = 100 x 100 (460 nonzero entries)
initial vector x = 0
precision : double
solver    : BiCG 2
precon    : none
storage   : CRS
lis_solve : normal end

BiCG: number of iterations      = 15 (double = 15, quad = 0)
BiCG: elapsed time              = 2.911400e-03 sec.
BiCG: preconditioner           = 1.560780e-04 sec.
BiCG: matrix creation          = 1.459997e-04 sec.
BiCG: linear solver            = 2.755322e-03 sec.
BiCG: relative residual 2-norm = 6.221213e-15
```

2.3.3 Installing

In the directory `lis-($VERSION)`, enter as follows:

```
>make install
```

It copies the files to the destination as follows:

```
$(INSTALLDIR)
+include
|   +lis_config.h lis.h lisf.h
+lib
    +liblis.a
```

`lis_config.h` is the header file required to build the library, and `lis.h` and `lisf.h` are the header files required by C and Fortran compilers, respectively. `liblis.a` is the library file.

2.4 Installing on Windows Systems

Use one of the solution files or the project files for Microsoft Visual Studio in the directory `lis-($VERSION)/win32`. `lis_with_fortran.sln` is the solution file to be used with Intel Visual Fortran Compiler. `lis_with_fortran_mpi.sln` is the solution file to be used with Visual Fortran and MPICH2. Header files are located in `lis-($VERSION)/include`. `lis_config_win32.h` is the header file required to build the library. `lis.h` and `lisf.h` are the header files required by C and Fortran compilers, respectively. The library files are generated in `lis-($VERSION)/lib`. The executable files of the test programs are generated in `lis-($VERSION)/test`.

2.5 Test Programs

2.5.1 test1

Usage: `test1 matrix_filename rhs_setting solution_filename residual_filename [options]`

This program inputs the data of the coefficient matrix from `matrix_filename` and solves the linear equation $Ax = b$ with the solver specified by `options`. It outputs the solution to `solution_filename` and the residual history to `residual_filename`. The Extended Matrix Market format, which is extended to allow vector data, is supported (see Appendix). One of the following values can be used for `rhs_setting`:

0	Use the right hand side vector b included in the data file
1	Use $b = (1, \dots, 1)^T$
2	Use $b = A \times (1, \dots, 1)^T$
<code>rhs_filename</code>	Filename for the right hand side vector

The PLAIN and Matrix Market formats are supported for `rhs_filename`. `test1f.F` is the Fortran version of `test1.c`.

2.5.2 test2

Usage: `test2 m n matrix_type solution_filename residual_filename [options]`

This program solves a discretized two dimensional Poisson equation $Ax = b$ using the five point central difference scheme, with the coefficient matrix A of size mn in the storage format specified by `matrix_type` and the solver specified by `options`. It outputs the solution to `solution_filename` and the residual history to `residual_filename`. The right hand side vector is set to make all the elements for the solution to be 1. The values m and n represent the numbers of grid points in each dimension.

2.5.3 test3

Usage: `test3 l m n matrix_type solution_filename residual_filename [options]`

This program solves a discretized three dimensional Poisson equation $Ax = b$ using the seven point central difference scheme, with the coefficient matrix A of size lmn in the storage format specified by `matrix_type` and the solver specified by `options`. It outputs the solution to `solution_filename` and the residual history to `residual_filename`. The right hand side vector is set to make all the elements for the solution to be 1. The values l , m and n represent the numbers of grid points in each dimension.

2.5.4 test4

This program solves the linear equation $Ax = b$ with a specified solver and a preconditioner, where A is a tridiagonal matrix

$$\begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

of size 12. The right hand side vector b is set to make all the elements of the solution x to be 1. `test4f.F` is the Fortran version of `test4.c`.

2.5.5 test5

Usage: test5 n gamma [options]

This program solves a linear equation $Ax = b$, where A is a Toeplitz matrix

$$\begin{pmatrix} 2 & 1 & & & & \\ 0 & 2 & 1 & & & \\ \gamma & 0 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & \gamma & 0 & 2 & 1 \\ & & & \gamma & 0 & 2 \end{pmatrix}$$

with the solver specified by **options**. Note that the right hand vector is set to make all the elements of the solution to be 1. The value n is the size of the matrix A .

2.5.6 etest1

Usage: etest1 matrix_filename solution_filename residual_filename [options]

This program inputs the matrix data from **matrix_filename** and solves the eigenvalue problem $Ax = \lambda x$ with the solver specified by **options**. It outputs the associated eigenvector to **solution_filename** and the residual history to **residual_filename**. The Matrix Market format is supported. **etest1f.F** is the Fortran version of **etest1.c**.

2.5.7 etest2

Usage: etest2 m n matrix_type solution_filename residual_filename [options]

This program solves the eigenvalue problem $Ax = \lambda x$, where the coefficient matrix A of size mn is derived from a discretized two dimensional Helmholtz equation using the five point central difference scheme, with the coefficient matrix in the storage format specified by **matrix_type** and the solver specified by **options**. It outputs the associated eigenvector to **solution_filename** and the residual history to **residual_filename**. The values m and n represent the numbers of grid points in each dimension.

2.5.8 etest3

Usage: etest3 l m n matrix_type solution_filename residual_filename [options]

This program solves the eigenvalue problem $Ax = \lambda x$, where the coefficient matrix A of size lmn is derived from a discretized three dimensional Helmholtz equation using the seven point central difference scheme, with the coefficient matrix in the storage format specified by **matrix_type** and the solver specified by **options**. It outputs the associated eigenvector to **solution_filename** and the residual history to **residual_filename**. The values l , m and n represent the numbers of grid points in each dimension.

2.5.9 etest4

Usage: etest4 n [options]

This program solves the eigenvalue problem $Ax = \lambda x$ with a specified solver, where A is a tridiagonal

matrix

$$A = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

of size $n \times n$. `etest4f.F` is the Fortran version of `etest4.c`.

2.5.10 etest5

Usage: `etest5 evaluate_filename evector_filename`

This program solves the eigenvalue problem $Ax = \lambda x$ with subspace iteration, where A is a tridiagonal matrix

$$A = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

of size 12×12 . It outputs 2 extreme eigenvalues of smallest magnitude to `evaluate_filename` and the associated eigenvectors to `evector_filename` in the extended Matrix Market format (see Appendix).

2.5.11 spmvtest1

Usage: `spmvtest1 n iter`

This program computes the multiply of a tridiagonal matrix derived from a discretized one dimensional Poisson equation using the three point central difference scheme

$$\begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

of size n and a vector $(1, \dots, 1)^T$. FLOPS performance is measured as the average of *iter* iterations.

2.5.12 spmvtest2

Usage: `spmvtest2 m n iter`

This program computes the multiply of a sparse matrix, derived from a discretized two dimensional Poisson equation using the five point central difference scheme with available matrix storage formats, and a vector $(1, \dots, 1)^T$. The values m and n represent the numbers of grid points in the vertical and horizontal directions. FLOPS performance is measured as the average of *iter* iterations.

2.5.13 spmvtest3

Usage: `spmvtest3 l m n iter`

This program computes the multiply of a sparse matrix, derived from a discretized three dimensional Poisson equation using the seven point central difference scheme with available matrix storage formats, and a vector $(1, \dots, 1)^T$. The values l , m and n represent the numbers of grid points in each dimension. FLOPS performance is measured as the average of *iter* iterations.

2.5.14 spmvtest4

Usage: `spmvtest4 matrix_filename_list iter [block]`

This program inputs the matrix data from the files listed in `matrix_filename_list`, and computes the multiplies of the matrices with available matrix storage formats and a vector $(1, \dots, 1)^T$. FLOPS performance is measured twice as the average of *iter* iterations. If necessary, the block size of BSR and BSC can be specified by *block*.

2.5.15 spmvtest5

Usage: `spmvtest5 matrix_filename matrix_type iter [block]`

This program inputs the matrix data from `matrix_filename` and compute the multiply of the matrix with `matrix_type` and a vector $(1, \dots, 1)^T$. FLOPS performance is measured twice as the average of *iter* iterations. If necessary, the block size of BSR and BSC can be specified by *block*.

2.6 Restrictions

The current version has the following restrictions:

- Preconditioners
 - If a preconditioner other than the Jacobi or SSOR is selected and the matrix A is not in the CRS format, a new matrix is created in the CRS format for preconditioning.
 - The SA-AMG preconditioner does not support the BiCG.
 - The SA-AMG preconditioner does not support the multithreaded environment.
 - The assembly of preconditioning matrices in the SAINV is not parallelized.
- Quadruple precision operations
 - The Jacobi, Gauss-Seidel, SOR, and IDR(s) methods do not support quadruple precision operations.
 - The conjugate gradient and conjugate residual methods for eigenproblems do not support quadruple precision operations.
 - The Jacobi, Gauss-Seidel and SOR do not support quadruple precision operations in the hybrid preconditioner.
 - The I+S and SA-AMG preconditioners do not support quadruple precision operations.
- Matrix storage formats
 - In the MPI environment, the CRS is the only accepted format for user defined arrays.

3 Basic Operations

This section describes how to use the library. A program requires the following statements:

- Initialization
- Matrix creation
- Vector creation
- Solver creation
- Value assignment for matrices and vectors
- Solver assignment
- Solver execution
- Finalization

In addition, it must include one of the following `include` statements:

- C `#include "lis.h"`
- Fortran `#include "lisf.h"`

When Lis is installed in `$(INSTALLDIR)`, `lis.h` and `lisf.h` are located in `$(INSTALLDIR)/include`.

3.1 Initializing and Finalizing

The functions for initializing and finalizing the execution environment must be called at the top and the bottom of the program, respectively, as follows:

```
C
1: #include "lis.h"
2: int main(int argc, char* argv[])
3: {
4:     lis_initialize(&argc, &argv);
5:     ...
6:     lis_finalize();
7: }
```

```
Fortran
1: #include "lisf.h"
2:     call lis_initialize(ierr)
3:     ...
4:     call lis_finalize(ierr)
```

Initializing

For initialization, the following functions are used:

- C `lis_initialize(int* argc, char** argv[])`
- Fortran subroutine `lis_initialize(integer ierr)`

This function initializes the MPI execution environment, and specifies options on the command line.

Finalizing

For finalization, the following functions are used:

- C `int lis_finalize()`
- Fortran subroutine `lis_finalize(integer ierr)`

3.2 Operating Vectors

Assume that the size of a vector v is $global_n$, and the size of each partial vector stored on $nprocs$ processing elements is $local_n$. If $global_n$ is divisible, then $local_n = global_n / nprocs$. For example, when the vector v is stored on two processing elements, as shown in Equation (3.1), $global_n$ and $local_n$ are 4 and 2, respectively.

$$v = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} \text{PE0} \\ \text{PE1} \end{matrix} \quad (3.1)$$

In the case of creating the vector v in Equation (3.1), the vector v itself is created for the serial and multithreaded environments, while the partial vectors are created and stored on a given number of processing elements for the MPI environment.

Programs to create the vector v are as follows, where the number of the processing elements for the MPI environment is assumed to be two:

C (for serial and multithreaded environments) —

```
1: int          i,n;
2: LIS_VECTOR   v;
3: n = 4;
4: lis_vector_create(0,&v);
5: lis_vector_set_size(v,0,n); /* or lis_vector_set_size(v,n,0); */
6:
7: for(i=0;i<n;i++)
8: {
9:     lis_vector_set_value(LIS_INS_VALUE,i,(double)i,v);
10: }
```

C (for MPI environment) —

```
1: int          i,n,is,ie;                /*or int  i,ln,is,ie;                */
2: LIS_VECTOR   v;                        /*
3: n = 4;                                  /*  ln = 2;                        */
4: lis_vector_create(MPI_COMM_WORLD,&v);
5: lis_vector_set_size(v,0,n);            /*  lis_vector_set_size(v,ln,0);    */
6: lis_vector_get_range(v,&is,&ie);
7: for(i=is;i<ie;i++)
8: {
9:     lis_vector_set_value(LIS_INS_VALUE,i,(double)i,v);
10: }
```

Fortran (for serial and multithreaded environments) —

```
1: integer      i,n
2: LIS_VECTOR   v
3: n = 4
4: call lis_vector_create(0,v,ierr)
5: call lis_vector_set_size(v,0,n,ierr)
6:
7: do i=1,n
8:     call lis_vector_set_value(LIS_INS_VALUE,i,DBLE(i),v,ierr)
9: enddo
```

Fortran (for MPI environment) —

```
1: integer      i,n,is,ie
2: LIS_VECTOR   v
3: n = 4
4: call lis_vector_create(MPI_COMM_WORLD,v,ierr)
5: call lis_vector_set_size(v,0,n,ierr)
6: call lis_vector_get_range(v,is,ie,ierr)
7: do i=is,ie-1
8:     call lis_vector_set_value(LIS_INS_VALUE,i,DBLE(i),v,ierr);
9: enddo
```

Declaring Variables

As the second line shows, the declaration is stated as follows:

```
LIS_VECTOR    v;
```

Creating Vectors

To create the vector v , the following functions are used:

- C `int lis_vector_create(LIS_Comm comm, LIS_VECTOR *v)`
- Fortran subroutine `lis_vector_create(LIS_Comm comm, LIS_VECTOR v, integer ierr)`

For the example program above, `comm` must be replaced with the MPI communicator. For the serial and multithreaded environments, the value for `comm` is ignored.

Assigning Sizes

To assign the size of a vector v , the following functions are used:

- C `int lis_vector_set_size(LIS_VECTOR v, int local_n, int global_n)`
- Fortran subroutine `lis_vector_create(integer local_n, integer global_n, LIS_Comm comm, LIS_VECTOR v, integer ierr)`

Either *local_n* or *global_n* must be provided.

In the case of the serial and multithreaded environments, *local_n* is equal to *global_n*. Therefore, both `lis_vector_set_size(v,n,0)` and `lis_vector_set_size(v,0,n)` create a vector of size n .

For the MPI environment, `lis_vector_set_size(v,n,0)` creates a partial vector of size n on each processing element. On the other hand, `lis_vector_set_size(v,0,n)` creates a partial vector of size m_p on the processing element p . The values of m_p are determined by the library.

Assigning Elements

To assign an element to the i -th row of the vector v , the following functions are used:

- C `int lis_vector_set(int flag, int i, LIS_SCALAR value, LIS_VECTOR v)`
- Fortran subroutine `lis_vector_set_value(int flag, int i, LIS_SCALAR value, LIS_VECTOR v, integer ierr)`

For the MPI environment, the i -th row of the global vector must be specified, instead of the i -th row of the partial vector. Either

`LIS_INS_VALUE : v[i] = value`, or

`LIS_ADD_VALUE : v[i] = v[i] + value`

must be provided for `flag`.

Duplicating Vectors

To create a vector which has the same information as for an existing vector, the following functions are used:

- C `int lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR *vout)`
- Fortran subroutine `lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR vout, integer ierr)`

This function does not copy the elements of the vector. To copy the elements as well, the following functions must be called after the above functions:

- C `int lis_vector_copy(LIS_VECTOR vsrc, LIS_VECTOR vdst)`
- Fortran subroutine `lis_vector_copy(LIS_VECTOR vsrc, LIS_VECTOR vdst, integer ierr)`

Destroying Vectors

To destroy a vector, the following functions are used:

- C `int lis_vector_destroy(LIS_VECTOR v)`
- Fortran subroutine `lis_vector_destroy(LIS_VECTOR v, integer ierr)`

3.3 Operating Matrices

Assume that the size of a matrix A is $global_n \times global_n$, and that the size of each row block of the matrix A stored on $nprocs$ processing elements is $local_n \times global_n$. If $global_n$ is divisible, then $local_n = global_n / nprocs$. For example, when the row block of the matrix A is stored on two processing elements, as shown in Equation (3.2), $global_n$ and $local_n$ are 4 and 2, respectively.

$$A = \begin{pmatrix} 2 & 1 & & \\ 1 & 2 & 1 & \\ & 1 & 2 & 1 \\ & & 1 & 2 \end{pmatrix} \begin{matrix} \text{PE0} \\ \\ \text{PE1} \\ \end{matrix} \quad (3.2)$$

A matrix in a specific storage format can be created in one of the following three ways:

Method 1: Defining Arrays in Specific Storage Format with Library Functions

In the case of creating the matrix A in Equation (3.2) in the CRS format, the matrix A itself is created for the serial and multithreaded environments, while the partial matrices are created and stored on the given number of processing elements for the MPI environment.

Programs to create the matrix A in the CRS format are as follows, where the number of the processing elements for the MPI environment is assumed to be two:

C (for serial and multithreaded environments)

```

1: int          i,n;
2: LIS_MATRIX   A;
3: n = 4;
4: lis_matrix_create(0,&A);
5: lis_matrix_set_size(A,0,n); /* or lis_matrix_set_size(A,n,0); */
6: for(i=0;i<n;i++) {
7:     if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
8:     if( i<n-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);
9:     lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
10: }
11: lis_matrix_set_type(A,LIS_MATRIX_CRS);
12: lis_matrix_assemble(A);

```


C (for MPI environment)

```
1: int          i,n,gn,is,ie;
2: LIS_MATRIX   A;
3: gn = 4;                      /* or n=2 */
4: lis_matrix_create(MPI_COMM_WORLD,&A);
5: lis_matrix_set_size(A,0,gn); /* lis_matrix_set_size(A,n,0); */
6: lis_matrix_get_size(A,&n,&gn);
7: lis_matrix_get_range(A,&is,&ie);
8: for(i=is;i<ie;i++) {
9:     if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
10:    if( i<gn-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);
11:    lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
12: }
13: lis_matrix_set_type(A,LIS_MATRIX_CRS);
14: lis_matrix_assemble(A);
```

Fortran (for serial and multithreaded environments)

```
1: integer      i,n
2: LIS_MATRIX   A
3: n = 4
4: call lis_matrix_create(0,A,ierr)
5: call lis_matrix_set_size(A,0,n,ierr)
6: do i=1,n
7:     if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0d0,A,ierr)
8:     if( i<n ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0d0,A,ierr)
9:     call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
10: enddo
11: call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
12: call lis_matrix_assemble(A,ierr)
```

Fortran (for MPI environment)

```
1: integer      i,n,gn,is,ie
2: LIS_MATRIX   A
3: gn = 4
4: call lis_matrix_create(MPI_COMM_WORLD,A,ierr)
5: call lis_matrix_set_size(A,0,gn,ierr)
6: call lis_matrix_get_size(A,n,gn,ierr)
7: call lis_matrix_get_range(A,is,ie,ierr)
8: do i=is,ie-1
9:     if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0d0,A,ierr)
10:    if( i<gn ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0d0,A,ierr)
11:    call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
12: enddo
13: call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
14: call lis_matrix_assemble(A,ierr)
```

Declaring Variables

As the second line shows, the declaration is stated as follows:

```
LIS_MATRIX   A;
```

Creating Matrices

To create the matrix *A*, the following functions are used:

- C `int lis_matrix_create(LIS_Comm comm, LIS_MATRIX *A)`
- Fortran subroutine `lis_matrix_create(LIS_Comm comm, LIS_MATRIX A, integer ierr)`

`comm` must be replaced with the MPI communicator. For the serial and multithreaded environments, the value for `comm` is ignored.

Assigning Sizes

To assign a size to the matrix A , the following functions are used:

- C `int lis_matrix_set_size(LIS_MATRIX A, int local_n, int global_n)`
- Fortran subroutine `lis_matrix_set_size(LIS_MATRIX A, integer local_n, integer global_n, integer ierr)`

Either *local_n* or *global_n* must be provided.

In the case of the serial and multithreaded environments, *local_n* = *global_n*. Therefore, both `lis_matrix_set_size(A,n,0)` and `lis_matrix_set_size(A,0,n)` create a matrix of size $n \times n$.

For the MPI environment, `lis_matrix_set_size(A,n,0)` creates a partial matrix of size $n \times N$ on each processing element, where N is the total sum of n . On the other hand, `lis_matrix_set_size(A,0,n)` creates a partial matrix of size $m_p \times n$ on the processing element p . The values of m_p are determined by the library.

Assigning Elements

To assign an element to the cell at the i -th row and the j -th column of the matrix A , the following functions are used:

- C `int lis_matrix_set_value(int flag, int i, int j, LIS_SCALAR value, LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_value(integer flag, integer i, integer j, LIS_SCALAR value, LIS_MATRIX A, integer ierr)`

For the MPI environment, the i -th row and the j -th column of the global matrix must be specified, instead of the i -th row and the j -th column of the partial matrix. Either

`LIS_INS_VALUE` : $A(i, j) = \text{value}$, or

`LIS_ADD_VALUE` : $A(i, j) = A(i, j) + \text{value}$

must be provided for the parameter `flag`.

Assigning Storage Formats

To assign a storage format to the matrix A , the following functions are used:

- C `int lis_matrix_set_type(LIS_MATRIX A, int matrix_type)`
- Fortran subroutine `lis_matrix_set_type(LIS_MATRIX A, int matrix_type, integer ierr)`

`matrix_type` of A is `LIS_MATRIX_CRS` when the matrix is created. The following storage formats are supported:

Storage formats		matrix_type
Compressed Row Storage	(CRS)	{LIS_MATRIX_CRS 1}
Compressed Column Storage	(CCS)	{LIS_MATRIX_CCS 2}
Modified Compressed Sparse Row	(MSR)	{LIS_MATRIX_MSR 3}
Diagonal	(DIA)	{LIS_MATRIX_DIA 4}
Ellpack-Itpack generalized diagonal	(ELL)	{LIS_MATRIX_ELL 5}
Jagged Diagonal	(JDS)	{LIS_MATRIX_JDS 6}
Block Sparse Row	(BSR)	{LIS_MATRIX_BSR 7}
Block Sparse Column	(BSC)	{LIS_MATRIX_BSC 8}
Variable Block Row	(VBR)	{LIS_MATRIX_VBR 9}
Dense	(DNS)	{LIS_MATRIX_DNS 10}
Coordinate	(COO)	{LIS_MATRIX_COO 11}

Assembling Matrices

After assigning elements and storage formats, the following functions must be called:

- C `int lis_matrix_assemble(LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_assemble(LIS_MATRIX A, integer ierr)`

`lis_matrix_assemble` assembles A into the storage format specified by `lis_matrix_set_type`.

Destroying Matrices

To destroy a matrix, the following functions are used:

- C `int lis_matrix_destroy(LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_destroy(LIS_MATRIX A, integer ierr)`

Method 2: Defining Arrays in Specified Storage Format Directly

In the case of creating the matrix A in Equation (3.2) in the CRS format, the matrix A itself is created for the serial and multithreaded environments, while the partial matrices are created and stored on the given number of processing elements for the MPI environment.

Programs to create the matrix A in the CRS format are as follows, where the number of the processing elements for the MPI environment is assumed to be two:

C (for serial and multithreaded environments)

```
1: int          i,k,n,nnz;
2: int          *ptr,*index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; nnz = 10; k = 0;
6: lis_matrix_malloc_crs(n,nnz,&ptr,&index,&value);
7: lis_matrix_create(0,&A);
8: lis_matrix_set_size(A,0,n); /* or lis_matrix_set_size(A,n,0); */
9:
10: for(i=0;i<n;i++)
11: {
12:     if( i>0 ) {index[k] = i-1; value[k] = 1; k++;}
13:     index[k] = i; value[k] = 2; k++;
14:     if( i<n-1 ) {index[k] = i+1; value[k] = 1; k++;}
15:     ptr[i+1] = k;
16: }
17: ptr[0] = 0;
18: lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

C (for MPI environment)

```
1: int          i,k,n,nnz,is,ie;
2: int          *ptr,*index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 2; nnz = 5; k = 0;
6: lis_matrix_malloc_crs(n,nnz,&ptr,&index,&value);
7: lis_matrix_create(MPI_COMM_WORLD,&A);
8: lis_matrix_set_size(A,n,0);
9: lis_matrix_get_range(A,&is,&ie);
10: for(i=is;i<ie;i++)
11: {
12:     if( i>0 ) {index[k] = i-1; value[k] = 1; k++;}
13:     index[k] = i; value[k] = 2; k++;
14:     if( i<n-1 ) {index[k] = i+1; value[k] = 1; k++;}
15:     ptr[i-is+1] = k;
16: }
17: ptr[0] = 0;
18: lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

Associating Arrays

To associate the arrays in the CRS format with the matrix A , the following functions are used:

- C `int lis_matrix_set_crs(int nnz, int row[], int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_crs(integer nnz, integer row(), integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

Method 3: Reading Matrix and Vector Data from External Files

Programs to read the matrix A in Equation (3.2) in the CRS format and vector b in Equation (3.1) from an external file are as follows:

C (for serial, multithreaded and MPI environments)

```
1: LIS_MATRIX    A;
2: LIS_VECTOR    b,x;
3: lis_matrix_create(LIS_COMM_WORLD,&A);
4: lis_vector_create(LIS_COMM_WORLD,&b);
5: lis_vector_create(LIS_COMM_WORLD,&x);
6: lis_matrix_set_type(A,LIS_MATRIX_CRS);
7: lis_input(A,b,x,"matvec.mtx");
```

Fortran (for serial, multithreaded and MPI environments)

```
1: LIS_MATRIX    A
2: LIS_VECTOR    b,x
3: call lis_matrix_create(LIS_COMM_WORLD,A,ierr)
4: call lis_vector_create(LIS_COMM_WORLD,b,ierr)
5: call lis_vector_create(LIS_COMM_WORLD,x,ierr)
6: call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
7: call lis_input(A,b,x,'matvec.mtx',ierr)
```

The content of the destination file `matvec.mtx` is as follows:

```
%%MatrixMarket matrix coordinate real general
4 4 10 1 0
1 2 1.0e+00
1 1 2.0e+00
2 3 1.0e+00
2 1 1.0e+00
2 2 2.0e+00
3 4 1.0e+00
3 2 1.0e+00
3 3 2.0e+00
4 4 2.0e+00
4 3 1.0e+00
1 0.0e+00
2 1.0e+00
3 2.0e+00
4 3.0e+00
```

Reading from External Files

To input the matrix data for A from an external file, the following functions are used:

- C `int lis_input_matrix(LIS_MATRIX A, char *filename)`
- Fortran subroutine `lis_input(LIS_MATRIX A, character filename, integer ierr)`

`filename` must be replaced with the file path. The following file formats are supported:

- Matrix Market format
- Harwell-Boeing format

To read the data for the matrix A and vectors b and x from external files, the following functions are used:

- C `int lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, char *filename)`

- Fortran subroutine `lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, character filename, integer ierr)`

`filename` must be replaced with the file path. The following file formats are supported:

- Extended Matrix Market format (extended to allow vector data)
- Harwell-Boeing format

3.4 Solving Linear Equations

A program to solve the linear equation $Ax = b$ with a specified solver is as follows:

C (for serial, multithreaded and MPI environments)

```
1: LIS_MATRIX A;
2: LIS_VECTOR b,x;
3: LIS_SOLVER solver;
4:
5: /* Create matrix and vector */
6:
7: lis_solver_create(&solver);
8: lis_solver_set_option("-i bicg -p none",solver);
9: lis_solver_set_option("-tol 1.0e-12",solver);
10: lis_solver(A,b,x,solver);
```

Fortran (for serial, multithreaded and MPI environments)

```
1: LIS_MATRIX A
2: LIS_VECTOR b,x
3: LIS_SOLVER solver
4:
5: /* Create matrix and vector */
6:
7: call lis_solver_create(solver,ierr)
8: call lis_solver_set_option('-i bicg -p none',solver,ierr)
9: call lis_solver_set_option('-tol 1.0e-12',solver,ierr)
10: call lis_solver(A,b,x,solver,ierr)
```

Creating Solvers

To create a solver, the following functions are used:

- C `int lis_solver_create(LIS_SOLVER *solver)`
- Fortran subroutine `lis_solver_create(LIS_SOLVER solver, integer ierr)`

Specifying Options

To specify options, the following functions are used:

- C `int lis_solver_set_option(char *text, LIS_SOLVER solver)`
- Fortran subroutine `lis_solver_set_option(character text, LIS_SOLVER solver, integer ierr)`

or

- C `int lis_solver_set_optionC(LIS_SOLVER solver)`

- Fortran subroutine `lis_solver_set_optionC(LIS_SOLVER solver, integer ierr)`

`lis_solver_set_optionC` is a function which sets the options specified on the command line, and pass them to `solver` when the user's program is run.

The table below shows the available command line options, where `-i {cg|1}` means `-i cg` or `-i 1` and `-maxiter [1000]` indicates that `-maxiter` defaults to 1,000.

Specifying Linear Solvers (Default: `-i bicg`)

Method	Option	Auxiliary Options	
CG	<code>-i {cg 1}</code>		
BiCG	<code>-i {bicg 2}</code>		
CGS	<code>-i {cgs 3}</code>		
BiCGSTAB	<code>-i {bicgstab 4}</code>		
BiCGSTAB(l)	<code>-i {bicgstabl 5}</code>	<code>-ell [2]</code>	Degree l
GPBiCG	<code>-i {gpbicg 6}</code>		
TFQMR	<code>-i {tfqmr 7}</code>		
Orthomin(m)	<code>-i {orthomin 8}</code>	<code>-restart [40]</code>	Restart value m
GMRES(m)	<code>-i {gmres 9}</code>	<code>-restart [40]</code>	Restart value m
Jacobi	<code>-i {jacobi 10}</code>		
Gauss-Seidel	<code>-i {gs 11}</code>		
SOR	<code>-i {sor 12}</code>	<code>-omega [1.9]</code>	Relaxation coefficient ω ($0 < \omega < 2$)
BiCGSafe	<code>-i {bicgsafe 13}</code>		
CR	<code>-i {cr 14}</code>		
BiCR	<code>-i {bicr 15}</code>		
CRS	<code>-i {crs 16}</code>		
BiCRSTAB	<code>-i {bicrstab 17}</code>		
GPBiCR	<code>-i {gpbicr 18}</code>		
BiCRSafe	<code>-i {bicrsafe 19}</code>		
FGMRES(m)	<code>-i {fgmres 20}</code>	<code>-restart [40]</code>	Restart value m
IDR(s)	<code>-i {idrs 21}</code>	<code>-irestart [2]</code>	Restart value s
MINRES	<code>-i {minres 22}</code>		

Specifying Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options
None	-p {none 0}	
Jacobi	-p {jacobi 1}	
ILU(k)	-p {ilu 2}	-ilu_fill [0] Fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0] Relaxation coefficient ω ($0 < \omega < 2$)
Hybrid	-p {hybrid 4}	-hybrid_i [sor] Linear equations solver -hybrid_maxiter [25] Maximum number of iterations -hybrid_tol [1.0e-3] Convergence criterion -hybrid_w [1.5] Relaxation coefficient ω for SOR ($0 < \omega < 2$) -hybrid_ell [2] Degree l of BiCGSTAB(l) -hybrid_restart [40] Restart values for GMRES and Orthomin
I+S	-p {is 5}	-is_alpha [1.0] Parameter α for preconditioner of $I + \alpha S^{(m)}$ type -is_m [3] Parameter m for preconditioner of $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05] Drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false] Selects unsymmetric version (Matrix structure must be symmetric) -saamg_theta [0.05 0.12] Drop criterion $a_{ij}^2 \leq \theta^2 a_{ii} a_{jj} $ (symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05] Drop criterion -iluc_rate [5.0] Ratio of maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05] Drop criterion -ilut_rate [5.0] Ratio of maximum fill-in
Additive Schwarz	-adds true	-adds_iter [1] Number of iterations

Other Options

Option	
-maxiter [1000]	Maximum number of iterations
-tol [1.0e-12]	Convergence criterion
-print [0]	Display of the residual
	-print {none 0} None
	-print {mem 1} Saves the residual history in memory
	-print {out 2} Displays the residual history
	-print {all 3} Saves the residual history and displays it on the screen
-scale [0]	Scaling (The result will overwrite the original matrix and vectors)
	-scale {none 0} No scaling
	-scale {jacobi 1} Jacobi scaling $D^{-1}Ax = D^{-1}b$ (D represents the diagonal of $A = (a_{ij})$)
	-scale {symm_diag 2} Diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$ ($D^{-1/2}$ represents a diagonal matrix with $1/\sqrt{a_{ii}}$ as diagonal)
-initx_zeros [true]	Behavior of the initial vector x_0
	-initx_zeros {false 0} Given values
	-initx_zeros {true 1} All elements are set to 0
-omp_num_threads [t]	Number of threads (t represents the maximum number of threads)
-storage [0]	Matrix storage format
-storage_block [2]	Block size of BSR and BSC

Precision (Default: -f double)

Precision	Option	Auxiliary Options
DOUBLE	-f {double 0}	
QUAD	-f {quad 1}	

Solving Linear Equations

To solve the linear equation $Ax = b$, the following functions are used:

- C `int lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver)`
- Fortran subroutine `lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver, integer ierr)`

3.5 Solving Eigenvalue Problems

A program to solve the eigenvalue problem $Ax = \lambda x$ with a specified solver is as follows:

C (for serial, multithreaded and MPI environments)

```
1: LIS_MATRIX A;
2: LIS_VECTOR x;
3: LIS_REAL eval;
4: LIS_ESOLVER esolver;
5:
6: /* Create matrix and vector */
7:
8: lis_esolver_create(&esolver);
9: lis_esolver_set_option("-e ii -i bicg -p none",esolver);
10: lis_esolver_set_option("-etol 1.0e-12 -tol 1.0e-12",esolver);
11: lis_solve(A,x,eval,esolver);
```

Fortran (for serial, multithreaded and MPI environments)

```
1: LIS_MATRIX A
2: LIS_VECTOR x
3: LIS_REAL eval
4: LIS_ESOLVER esolver
5:
6: /* Create matrix and vector */
7:
8: call lis_esolver_create(esolver,ierr)
9: call lis_esolver_set_option('e ii -i bicg -p none',esolver,ierr)
10: call lis_esolver_set_option('etol 1.0e-12 -tol 1.0e-12',esolver,ierr)
11: call lis_solve(A,x,eval,esolver,ierr)
```

Creating Eigensolvers

To create an eigensolver, the following functions are used:

- C `int lis_esolver_create(LIS_ESOLVER *esolver)`
- Fortran subroutine `lis_esolver_create(LIS_ESOLVER esolver, integer ierr)`

Specifying Options

To specify options, the following functions are used:

- C `int lis_esolver_set_option(char *text, LIS_ESOLVER esolver)`
- Fortran subroutine `lis_esolver_set_option(character text, LIS_ESOLVER esolver, integer ierr)`

or

- C `int lis_esolver_set_optionC(LIS_ESOLVER esolver)`
- Fortran subroutine `lis_esolver_set_optionC(LIS_ESOLVER esolver, integer ierr)`

`lis_esolver_set_optionC` is a function which sets the options specified on the command line, and pass them to `esolver` when the user's program is run.

The table below shows the available command line options, where `-e {pi|1}` means `-e pi` or `-e 1` and `-emaxiter [1000]` indicates that `-emaxiter` defaults to 1,000.

Specifying Eigensolvers (Default: `-i bicg`)

Method	Option	Auxiliary Options	
Power Iteration	<code>-e {pi 1}</code>		
Inverse Iteration	<code>-e {ii 2}</code>	<code>-i [bicg]</code>	Linear solver
Approximate Inverse Iteration	<code>-e {aii 3}</code>		
Rayleigh Quotient Iteration	<code>-e {rqi 4}</code>	<code>-i [bicg]</code>	Linear solver
Subspace Iteration	<code>-e {si 5}</code>	<code>-ss [2]</code>	Size of subspace
		<code>-m [0]</code>	Mode number
Lanczos Iteration	<code>-e {li 6}</code>	<code>-ss [2]</code>	Size of subspace
		<code>-m [0]</code>	Mode number
Conjugate Gradient	<code>-e {cg 7}</code>		
Conjugate Residual	<code>-e {cr 8}</code>		

Specifying Preconditioners (Default: `-p ilu`)

Preconditioner	Option	Auxiliary Options	
None	<code>-p {none 0}</code>		
Jacobi	<code>-p {jacobi 1}</code>		
ILU(k)	<code>-p {ilu 2}</code>	<code>-ilu_fill [0]</code>	Fill level k
SSOR	<code>-p {ssor 3}</code>	<code>-ssor_w [1.0]</code>	Relaxation coefficient ω ($0 < \omega < 2$)
Hybrid	<code>-p {hybrid 4}</code>	<code>-hybrid_i [sor]</code>	Linear equations solver
		<code>-hybrid_maxiter [25]</code>	Maximum number of iterations
		<code>-hybrid_tol [1.0e-3]</code>	Convergence criterion
		<code>-hybrid_w [1.5]</code>	Relaxation coefficient ω for SOR ($0 < \omega < 2$)
		<code>-hybrid_ell [2]</code>	Degree l of BiCGSTAB(l)
		<code>-hybrid_restart [40]</code>	Restart values for GMRES and Orthomin
I+S	<code>-p {is 5}</code>	<code>-is_alpha [1.0]</code>	Parameter α for preconditioner of $I + \alpha S^{(m)}$ type
		<code>-is_m [3]</code>	Parameter m for preconditioner of $I + \alpha S^{(m)}$ type
SAINV	<code>-p {sainv 6}</code>	<code>-sainv_drop [0.05]</code>	Drop criterion
SA-AMG	<code>-p {saamg 7}</code>	<code>-saamg_unsym [false]</code>	Selects unsymmetric version (Matrix structure must be symmetric)
		<code>-saamg_theta [0.05 0.12]</code>	Drop criterion $a_{ij}^2 \leq \theta^2 a_{ii} a_{jj} $ (symmetric or unsymmetric)
Crout ILU	<code>-p {iluc 8}</code>	<code>-iluc_drop [0.05]</code>	Drop criterion
		<code>-iluc_rate [5.0]</code>	Ratio of maximum fill-in
ILUT	<code>-p {ilut 9}</code>	<code>-ilut_drop [0.05]</code>	Drop criterion
		<code>-ilut_rate [5.0]</code>	Ratio of maximum fill-in
Additive Schwarz	<code>-adds true</code>	<code>-adds_iter [1]</code>	Number of iterations

Other Options

Option	
<code>-emaxiter [1000]</code>	Maximum number of iterations
<code>-etol [1.0e-12]</code>	Convergence criterion
<code>-eprint [0]</code>	Display of the residual
	<code>-eprint {none 0}</code> None
	<code>-eprint {mem 1}</code> Saves the residual history in memory
	<code>-eprint {out 2}</code> Displays the residual history
	<code>-eprint {all 3}</code> Saves the residual history and displays it on the screen
<code>-ie [ii]</code>	Inner eigensolver used in Lanczos Iteration or Subspace Iteration
	<code>-ie {pi 1}</code> Power Iteration (Subspace Iteration only)
	<code>-ie {ii 2}</code> Inverse Iteration
	<code>-ie {aii 3}</code> Approximate Inverse Iteration
	<code>-ie {rqi 4}</code> Rayleigh Quotient Iteration
<code>-shift [0.0]</code>	Amount of shift
<code>-initx_ones [true]</code>	Behavior of the initial vector x_0
	<code>-initx_ones {false 0}</code> Given values
	<code>-initx_ones {true 1}</code> All elements are set to 1
<code>-omp_num_threads [t]</code>	Number of threads (t represents the maximum number of threads)
<code>-estorage [0]</code>	Matrix storage format
<code>-estorage_block [2]</code>	Block size of BSR and BSC

Precision (Default: `-ef double`)

Precision	Option	Auxiliary Options
DOUBLE	<code>-ef {double 0}</code>	
QUAD	<code>-ef {quad 1}</code>	

Solving Eigenvalue Problems

To solve the eigenvalue problem $Ax = \lambda x$, the following functions are used:

- C `int lis_solve(LIS_MATRIX A, LIS_VECTOR x, LIS_REAL eval, LIS_ESOLVER solver)`
- Fortran subroutine `lis_solve(LIS_MATRIX A, LIS_VECTOR x, LIS_REAL eval, LIS_ESOLVER solver, integer ierr)`

3.6 Sample Programs

The following are the programs for solving the linear equation $Ax = b$, where the matrix A is a tridiagonal matrix

$$\begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

of size 12. The the right hand side vector b is set to make all the elements of the solution x is 1. The program is located in the directory `lis-($VERSION)/test`.

Test program: test4.c

```
1: #include <stdio.h>
2: #include "lis.h"
3: main(int argc, char *argv[])
4: {
5:     int i,n,gm,is,ie,iter;
6:     LIS_MATRIX A;
7:     LIS_VECTOR b,x,u;
8:     LIS_SOLVER solver;
9:     n = 12;
10:    lis_initialize(&argc,&argv);
11:    lis_matrix_create(LIS_COMM_WORLD,&A);
12:    lis_matrix_set_size(A,0,n);
13:    lis_matrix_get_size(A,&n,&gm)
14:    lis_matrix_get_range(A,&is,&ie)
15:    for(i=is;i<ie;i++)
16:    {
17:        if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,-1.0,A);
18:        if( i<gm-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,-1.0,A);
19:        lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
20:    }
21:    lis_matrix_set_type(A,LIS_MATRIX_CRS);
22:    lis_matrix_assemble(A);
23:
24:    lis_vector_duplicate(A,&u);
25:    lis_vector_duplicate(A,&b);
26:    lis_vector_duplicate(A,&x);
27:    lis_vector_set_all(1.0,u);
28:    lis_matvec(A,u,b);
29:
30:    lis_solver_create(&solver);
31:    lis_solver_set_optionC(solver);
32:    lis_solve(A,b,x,solver);
33:    lis_solver_get_iters(solver,&iter);
34:    printf("iter = %d\n",iter);
35:    lis_vector_print(x);
36:    lis_matrix_destroy(A);
37:    lis_vector_destroy(u);
38:    lis_vector_destroy(b);
39:    lis_vector_destroy(x);
40:    lis_solver_destroy(solver);
41:    lis_finalize();
42:    return 0;
43: }
```

Test program: test4f.F

```
1:      implicit none
2:
3: #include "lisf.h"
4:
5:      integer          i,n,gn,is,ie,iter,ierr
6:      LIS_MATRIX       A
7:      LIS_VECTOR       b,x,u
8:      LIS_SOLVER       solver
9:      n = 12
10:     call lis_initialize(ierr)
11:     call lis_matrix_create(LIS_COMM_WORLD,A,ierr)
12:     call lis_matrix_set_size(A,0,n,ierr)
13:     call lis_matrix_get_size(A,n,gn,ierr)
14:     call lis_matrix_get_range(A,is,ie,ierr)
15:     do i=is,ie-1
16:         if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,-1.0d0,
17:                                             A,ierr)
18:         if( i<gn ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,-1.0d0,
19:                                             A,ierr)
20:         call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
21:     enddo
22:     call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
23:     call lis_matrix_assemble(A,ierr)
24:
25:     call lis_vector_duplicate(A,u,ierr)
26:     call lis_vector_duplicate(A,b,ierr)
27:     call lis_vector_duplicate(A,x,ierr)
28:     call lis_vector_set_all(1.0d0,u,ierr)
29:     call lis_matvec(A,u,b,ierr)
30:
31:     call lis_solver_create(solver,ierr)
32:     call lis_solver_set_optionC(solver,ierr)
33:     call lis_solve(A,b,x,solver,ierr)
34:     call lis_solver_get_iters(solver,iter,ierr)
35:     write(*,*) 'iter = ',iter
36:     call lis_vector_print(x,ierr)
37:     call lis_matrix_destroy(A,ierr)
38:     call lis_vector_destroy(b,ierr)
39:     call lis_vector_destroy(x,ierr)
40:     call lis_vector_destroy(u,ierr)
41:     call lis_solver_destroy(solver,ierr)
42:     call lis_finalize(ierr)
43:
44:     stop
45:     end
```

3.7 Compiling and Linking

Provided below is an example `test4.c` located in the directory `lis-($VERSION)/test`, compiled on SGI Altix 3700 using Intel C/C++ Compiler 8.1 (icc). Since the library includes Fortran 90 codes when the SA-AMG preconditioner is used, linking is processed by a Fortran 90 compiler.

for serial environment

Compiling

```
>icc -c -I$(INSTALLDIR)/include test4.c
```

Linking

```
>icc -o test4 test4.o -llis
```

Linking (with SA-AMG)

```
>ifort -nofor_main -o test4 test4.o -llis
```

for multithreaded environment

Compiling

```
>icc -c -openmp -I$(INSTALLDIR)/include test4.c
```

Linking

```
>icc -openmp -o test4 test4.o -llis
```

Linking (with SA-AMG)

```
>ifort -nofor_main -openmp -o test4 test4.o -llis
```

for MPI environment

Compiling

```
>icc -c -DUSE_MPI -I$(INSTALLDIR)/include test4.c
```

Linking

```
>icc -o test4 test4.o -llis -lmpi
```

Linking (with SA-AMG)

```
>ifort -nofor_main -o test4 test4.o -llis -lmpi
```

for multithreaded MPI environment

Compiling

```
>icc -c -openmp -DUSE_MPI -I$(INSTALLDIR)/include test4.c
```

Linking

```
>icc -openmp -o test4 test4.o -llis -lmpi
```

Linking (with SA-AMG)

```
>ifort -nofor_main -openmp -o test4 test4.o -llis -lmpi
```

Provided below is an example `test4f.F` located in the directory `lis-($VERSION)/test`, compiled on SGI Altix 3700 using Intel Fortran Compiler 8.1 (ifort). Since an `#include` statement is used in the program, a compiler option `-fpp` is specified to use the preprocessor.

for serial environment

Compiling

```
>ifort -c -fpp -I$(INSTALLDIR)/include test4f.F
```

Linking

```
>ifort -o test4 test4.o -llis
```

for multithreaded environment

Compiling

```
>ifort -c -fpp -openmp -I$(INSTALLDIR)/include test4f.F
```

Linking

```
>ifort -openmp -o test4 test4.o -llis
```

for MPI environment

Compiling

```
>ifort -c -fpp -DUSE_MPI -I$(INSTALLDIR)/include test4f.F
```

Linking

```
>ifort -o test4 test4.o -llis -lmpi
```

for multithreaded MPI environment

Compiling

```
>ifort -c -fpp -openmp -DUSE_MPI -I$(INSTALLDIR)/include test4f.F
```

Linking

```
>ifort -openmp -o test4 test4.o -llis -lmpi
```

3.8 Running

The test programs `test4` and `test4f` in the directory `lis-($VERSION)/test` are run as follows:

for serial environment

```
>./test4 -i bicgstab
```

for multithreaded environment

```
>env OMP_NUM_THREADS=2 ./test4 -i bicgstab
```

for MPI environment

```
>mpirun -np 2 ./test4 -i bicgstab
```

for multithreaded MPI environment

```
>mpirun -np 2 env OMP_NUM_THREADS=2 ./test4 -i bicgstab
```

The following results will be returned:

```
precision : double
solver     : BiCGSTAB 4
precon     : none
storage    : CRS
lis_solve  : normal end
```

```
iter = 6
 0 1.000000e+000
 1 1.000000e+000
 2 1.000000e+000
 3 1.000000e+000
 4 1.000000e+000
 5 1.000000e+000
 6 1.000000e+000
 7 1.000000e+000
 8 1.000000e+000
 9 1.000000e+000
10 1.000000e+000
11 1.000000e+000
```

4 Quadruple Precision Operations

Double precision operations sometimes require a large number of iterations because of the rounding error. Lis supports "double-double", or quadruple precision operations by combining two double precision floating point numbers[15, 16]. To use the quadruple precision with the same interface as the double precision operations, both the matrix and vectors are assumed to be double precision. Lis also supports the performance acceleration of quadruple precision operations with SIMD instructions, such as Intel's Streaming SIMD Extensions (SSE) and IBM's Fused Multiply-Add (FMA)[24].

4.1 Using Quadruple Precision Operations

The test program `test5.c` solves a linear equation $Ax = b$, where A is a Toeplitz matrix

$$\begin{pmatrix} 2 & 1 & & & & \\ 0 & 2 & 1 & & & \\ \gamma & 0 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & \gamma & 0 & 2 & 1 \\ & & & \gamma & 0 & 2 \end{pmatrix}.$$

The right hand vector is set to make all the elements of the solution to be 1. The value n is the size of the matrix A . `test5` with `-f` option is run as follows:

Double precision

By entering `>./test5 200 2.0 -f double`
the following results will be returned:

```
n = 200, gamma = 2.000000
initial vector x = 0
precision : double
solver    : BiCG 2
precon    : none
storage   : CRS
lis_solve : LIS_MAXITER(code=4)

BiCG: number of iterations      = 1001 (double = 1001, quad = 0)
BiCG: elapsed time              = 2.044368e-02 sec.
BiCG: preconditioner           = 4.768372e-06 sec.
BiCG: matrix creation           = 4.768372e-06 sec.
BiCG: linear solver             = 2.043891e-02 sec.
BiCG: relative residual 2-norm = 8.917591e+01
```

Quadruple precision

By entering `>./test5 200 2.0 -f quad`
the following results will be returned:

```
n = 200, gamma = 2.000000
initial vector x = 0
precision : quad
solver    : BiCG 2
precon    : none
storage   : CRS
lis_solve : normal end
```



```
BiCG: number of iterations      = 230 (double = 230, quad = 0)
BiCG: elapsed time              = 2.267408e-02 sec.
BiCG:   preconditioner          = 4.549026e-04 sec.
BiCG:     matrix creation        = 5.006790e-06 sec.
BiCG:    linear solver           = 2.221918e-02 sec.
BiCG: relative residual 2-norm = 6.499145e-11
```

5 Matrix Storage Formats

This section describes the matrix storage formats supported by the library. Assume that the matrix row (column) number begins with 0 and that the number of the nonzero elements of the matrix A of $n \times n$ is nnz .

5.1 Compressed Row Storage (CRS)

The CRS format uses three arrays `ptr`, `index` and `value` to store data.

- `value` is a double precision array with a length of nnz , which stores the nonzero elements of the matrix A along the row.
- `index` is an integer array with a length of nnz , which stores the column numbers of the nonzero elements stored in the array `value`.
- `ptr` is an integer array with a length of $n + 1$, which stores the starting points of the rows of the arrays `value` and `index`.

5.1.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 2 shows how the matrix A in Figure 2 is stored in the CRS format. A program to create the matrix in the CRS format is as follows:

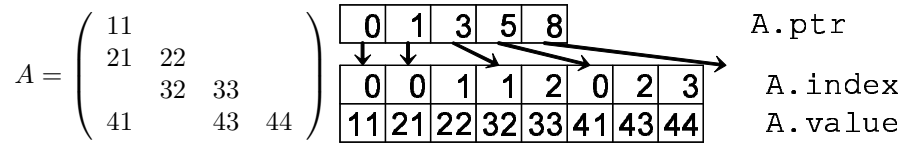


Figure 2: The data structure of the CRS format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int          n,nnz;
2: int          *ptr,*index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; nnz = 8;
6: ptr = (int *)malloc( (n+1)*sizeof(int) );
7: index = (int *)malloc( nnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: ptr[0] = 0; ptr[1] = 1; ptr[2] = 3; ptr[3] = 5; ptr[4] = 8;
13: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 1;
14: index[4] = 2; index[5] = 0; index[6] = 2; index[7] = 3;
15: value[0] = 11; value[1] = 21; value[2] = 22; value[3] = 32;
16: value[4] = 33; value[5] = 41; value[6] = 43; value[7] = 44;
17:
18: lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);

```

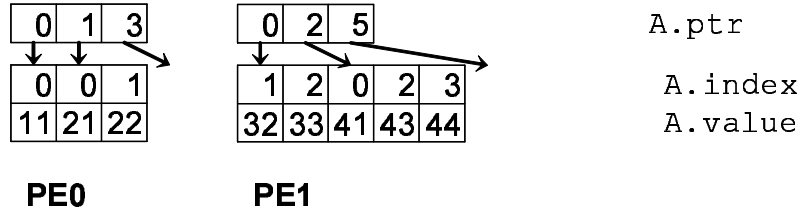


Figure 3: The data structure of the CRS format (for MPI environment).

5.1.2 Creating Matrices (for MPI Environment)

Figure 3 shows how the matrix A in Figure 2 is stored in the CRS format on two processing elements. A program to create the matrix in the CRS format on two processing elements is as follows:

for MPI environment

```

1: int      i,k,n,nnz,my_rank;
2: int      *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else          {n = 2; nnz = 5;}
8: ptr  = (int *)malloc( (n+1)*sizeof(int) );
9: index = (int *)malloc( nnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:     ptr[0] = 0; ptr[1] = 1; ptr[2] = 3;
15:     index[0] = 0; index[1] = 0; index[2] = 1;
16:     value[0] = 11; value[1] = 21; value[2] = 22;}
17: else {
18:     ptr[0] = 0; ptr[1] = 2; ptr[2] = 5;
19:     index[0] = 1; index[1] = 2; index[2] = 0; index[3] = 2; index[4] = 3;
20:     value[0] = 32; value[1] = 33; value[2] = 41; value[3] = 43; value[4] = 44;}
21: lis_matrix_set_crs(nnz,ptr,index,value,A);
22: lis_matrix_assemble(A);

```

5.1.3 Associating Arrays

To associate the arrays in the CRS format with the matrix A , the following functions are used:

- C `int lis_matrix_set_crs(int nnz, int row[], int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_crs(integer nnz, integer row(), integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.2 Compressed Column Storage (CCS)

The CSS format uses three arrays `ptr`, `index` and `value` to store data.

- `value` is a double precision array with a length of nnz , which stores the values for the nonzero elements of the matrix A along the column.
- `index` is an integer array with a length of nnz , which stores the row numbers of the nonzero elements stored in the array `value`.
- `ptr` is an integer array with a length of $n + 1$, which stores the starting points of the rows of the arrays `value` and `index`.

5.2.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 4 shows how the matrix A in Figure 4 is stored in the CCS format. A program to create the matrix in the CCS format is as follows:

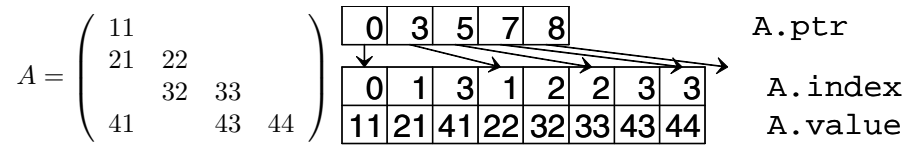


Figure 4: The data structure of the CCS format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int      n,nnz;
2: int      *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8;
6: ptr = (int *)malloc( (n+1)*sizeof(int) );
7: index = (int *)malloc( nnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: ptr[0] = 0; ptr[1] = 3; ptr[2] = 5; ptr[3] = 7; ptr[4] = 8;
13: index[0] = 0; index[1] = 1; index[2] = 3; index[3] = 1;
14: index[4] = 2; index[5] = 2; index[6] = 3; index[7] = 3;
15: value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22;
16: value[4] = 32; value[5] = 33; value[6] = 43; value[7] = 44;
17:
18: lis_matrix_set_ccs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);

```

5.2.2 Creating Matrices (for MPI Environment)

Figure 5 shows how the matrix A in Figure 4 is stored on two processing elements. A program to create the matrix in the CCS format on two processing elements is as follows:

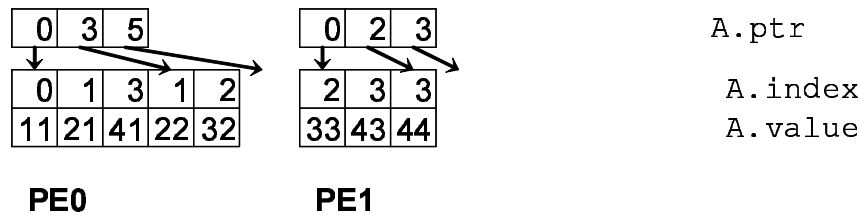


Figure 5: The data structure of the CCS format (for MPI environment).

for MPI environment

```

1: int      i,k,n,nnz,my_rank;
2: int      *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else      {n = 2; nnz = 5;}
8: ptr = (int *)malloc( (n+1)*sizeof(int) );
9: index = (int *)malloc( nnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:     ptr[0] = 0; ptr[1] = 3; ptr[2] = 5;
15:     index[0] = 0; index[1] = 1; index[2] = 3; index[3] = 1; index[4] = 2;
16:     value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22; value[4] = 32;
17: } else {
18:     ptr[0] = 0; ptr[1] = 2; ptr[2] = 3;
19:     index[0] = 2; index[1] = 3; index[2] = 3;
20:     value[0] = 33; value[1] = 43; value[2] = 44;
21:     lis_matrix_set_ccs(nnz,ptr,index,value,A);
22:     lis_matrix_assemble(A);

```

5.2.3 Associating Arrays

To associate the arrays in the CCS format with the matrix A , the following functions are used:

- C `int lis_matrix_set_ccs(int nnz, int row[], int index[], LIS_SCALAR value[],`
- Fortran subroutine `lis_matrix_set_ccs(integer nnz, integer row(), integer index(),`
`LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.3 Modified Compressed Sparse Row (MSR)

The MSR format is a modified version of the CRS format. The MSR format is different in that it separates the diagonal elements before storing it. The MSR format uses two arrays `index` and `value` to store data. Assume that `ndz` represents the number of the zero elements of the diagonal.

- `value` is a double precision array with a length of $nnz + ndz + 1$, which stores the diagonal of the matrix A down to the n -th element. The $n + 1$ -th element is not used. For the $n + 2$ -th and after, the values of the nonzero elements except the diagonal of the matrix A are stored along the row.
- `index` is an integer array with a length of $nnz + ndz + 1$, which stores the starting points of the rows of the off-diagonal elements of the matrix A down to the $n + 1$ -th element. For the $n + 2$ -th and after, it stores the row numbers of the off-diagonal elements of the matrix A stored in the array `value`.

5.3.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 6 shows how matrix A is stored in the MSR format. A program to create the matrix in the MSR format is as follows:

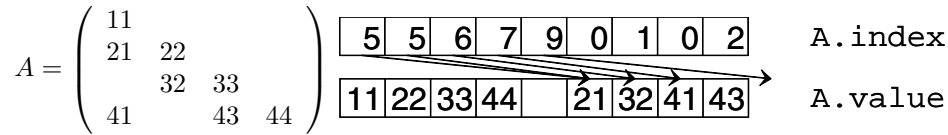


Figure 6: The data structure of the MSR format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int      n,nnz,ndz;
2: int      *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8; ndz = 0;
6: index = (int *)malloc( (nnz+ndz+1)*sizeof(int) );
7: value = (LIS_SCALAR *)malloc( (nnz+ndz+1)*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = 5; index[1] = 5; index[2] = 6; index[3] = 7;
12: index[4] = 9; index[5] = 0; index[6] = 1; index[7] = 0; index[8] = 2;
13: value[0] = 11; value[1] = 22; value[2] = 33; value[3] = 44;
14: value[4] = 0; value[5] = 21; value[6] = 32; value[7] = 41; value[8] = 43;
15:
16: lis_matrix_set_msr(nnz,ndz,index,value,A);
17: lis_matrix_assemble(A);

```

5.3.2 Creating Matrices (for MPI Environment)

Figure 7 shows how the matrix A in Figure 6 is stored in the MSR format on two processing elements. A program to create the matrix in the MSR format on two processing element is as follows:

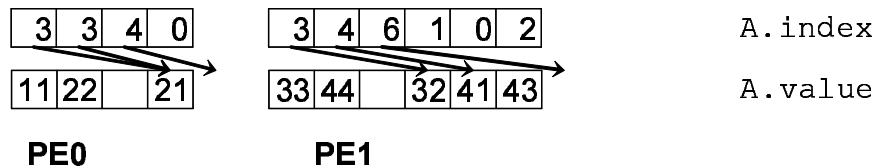


Figure 7: The data structure of the MSR format (for MPI environment).

for MPI environment

```

1: int          i,k,n,nnz,ndz,my_rank;
2: int          *index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3; ndz = 0;}
7: else          {n = 2; nnz = 5; ndz = 0;}
8: index = (int *)malloc( (nnz+ndz+1)*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( (nnz+ndz+1)*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:     index[0] = 3; index[1] = 3; index[2] = 4; index[3] = 0;
14:     value[0] = 11; value[1] = 22; value[2] = 0; value[3] = 21;}
15: else {
16:     index[0] = 3; index[1] = 4; index[2] = 6; index[3] = 1;
17:     index[4] = 0; index[5] = 2;
18:     value[0] = 33; value[1] = 44; value[2] = 0; value[3] = 32;
19:     value[4] = 41; value[5] = 43;}
20: lis_matrix_set_msr(nnz,ndz,index,value,A);
21: lis_matrix_assemble(A);

```

5.3.3 Associating Arrays

To associate the arrays in the MSR format with the matrix A , the following functions are used:

- C `int lis_matrix_set_msr(int nnz, int ndz, int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_msr(integer nnz, integer ndz, integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.4 Diagonal (DIA)

the DIA format uses two arrays `index` and `value` to store data. Assume that nnd represents the number of the nonzero diagonal elements of the matrix A .

- `value` is a double precision array with a length of $nnd \times n$, which stores nonzero diagonal elements of the matrix A .
- `index` is an integer array with a length of nnd , which stores the offsets from the main diagonal.

For the multithreaded environment, the following modifications have been made: the DIA format uses two arrays `index` and `value` to store data. Assume that $nprocs$ represents the number of the threads. nnd_p is the number of the nonzero diagonal elements of the partial matrix into which the row block of the matrix A is divided. $maxnnd$ is the maximum value nnd_p .

- `value` is a double precision array with a length of $maxnnd \times n$, which stores nonzero diagonal elements of the matrix A .
- `index` is an integer array with a length of $nprocs \times maxnnd$, which stores the offsets from the main diagonal.

5.4.1 Creating Matrices (for Serial Environment)

The right diagram in Figure 8 shows how the matrix A in Figure 8 is stored in the DIA format. A program to create the matrix in the DIA format is as follows:

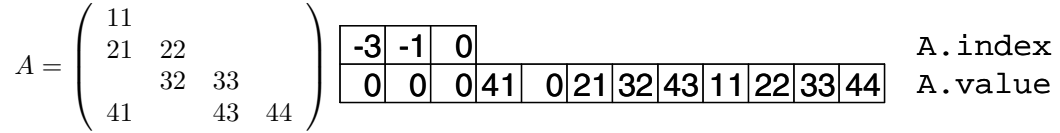


Figure 8: The data structure of the DIA format (for serial environment).

for serial environment

```

1: int      n,nnd;
2: int      *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnd = 3;
6: index = (int *)malloc( nnd*sizeof(int) );
7: value = (LIS_SCALAR *)malloc( n*nnd*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = -3; index[1] = -1; index[2] = 0;
12: value[0] = 0; value[1] = 0; value[2] = 0; value[3] = 41;
13: value[4] = 0; value[5] = 21; value[6] = 32; value[7] = 43;
14: value[8] = 11; value[9] = 22; value[10] = 33; value[11] = 44;
15:
16: lis_matrix_set_dia(nnd,index,value,A);
17: lis_matrix_assemble(A);

```


Figure 9 shows how the matrix A in Figure 8 is stored in the DIA format on two threads. A program to create the matrix in the DIA format on two threads is as follows:

Figure 9: The data structure of the DIA format (for multithreaded environment).

```

1: int          n,maxnnd,nprocs;
2: int          *index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; maxnnd = 3; nprocs = 2;
6: index = (int *)malloc( maxnnd*sizeof(int) );
7: value = (LIS_SCALAR *)malloc( n*maxnnd*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = -1; index[1] = 0; index[2] = 0; index[3] = -3; index[4] = -1; index[5] = 0;
12: value[0] = 0; value[1] = 21; value[2] = 11; value[3] = 22; value[4] = 0; value[5] = 0;
13: value[6] = 0; value[7] = 41; value[8] = 32; value[9] = 43; value[10] = 33; value[11] = 44;
14:
15: lis_matrix_set_dia(maxnnd,index,value,A);
16: lis_matrix_assemble(A);

```

5.4.3 Creating Matrices (for MPI Environment)

Figure 10 shows how the matrix A in Figure 8 is stored in the DIA format on two processing elements. A program to create the matrix in the DIA format on two processing elements is as follows:

<table><tr><td>-1</td><td>0</td></tr><tr><td>0</td><td>21</td></tr></table>	-1	0	0	21	<table><tr><td>-3</td><td>-1</td><td>0</td></tr><tr><td>0</td><td>41</td><td>32</td></tr></table>	-3	-1	0	0	41	32	A.index
-1	0											
0	21											
-3	-1	0										
0	41	32										
<table><tr><td>11</td><td>22</td></tr></table>	11	22	<table><tr><td>43</td><td>33</td><td>44</td></tr></table>	43	33	44	A.value					
11	22											
43	33	44										
PE0	PE1											

Figure 10: The data structure of the DIA format (for MPI environment).

for MPI environment

```

1: int          i,n,nnd,my_rank;
2: int          *index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnd = 2;}
7: else          {n = 2; nnd = 3;}
8: index = (int *)malloc( nnd*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( n*nnd*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:     index[0] = -1; index[1] = 0;
14:     value[0] = 0; value[1] = 21; value[2] = 11; value[3] = 22;}
15: else {
16:     index[0] = -3; index[1] = -1; index[2] = 0;
17:     value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 43; value[4] = 33;
18:     value[5] = 44;}
19: lis_matrix_set_dia(nnd,index,value,A);
20: lis_matrix_assemble(A);

```

5.4.4 Associating Arrays

To associate the arrays in the DIA format with the matrix A , the following functions are used:

- C `int lis_matrix_set_dia(int nnd, int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_dia(integer nnd, integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.5 Ellpack-Itpack generalized diagonal (ELL)

the ELL format uses two arrays `index` and `value` to store data. Assume that `maxnzc` is the maximum value for the number of the nonzero elements in the rows of the matrix A .

- `value` is a double precision array with a length of $\text{maxnzc} \times n$, which stores the nonzero elements of the rows of the matrix A along the column. The first column consists of the first nonzero elements of each row. If there is no nonzero elements to be stored, then 0 is stored.
- `index` is an integer array with a length of $\text{maxnzc} \times n$, which stores the column numbers of the nonzero elements stored in the array `value`. If the number of the nonzero elements in the i -th row is nnz , then `index[$nnz \times n + i$]` stores row number i .

5.5.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 11 shows how the matrix A in Figure 11 is stored in the ELL format. A program to create the matrix in the ELL format is as follows:

$$A = \begin{pmatrix} 11 & & & & \\ 21 & 22 & & & \\ & 32 & 33 & & \\ 41 & & 43 & 44 & \end{pmatrix} \quad \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 1 & 0 & 0 & 1 & 2 & 2 & 0 & 1 & 2 & 3 \\ \hline 11 & 21 & 32 & 41 & 0 & 22 & 33 & 43 & 0 & 0 & 0 & 44 \\ \hline \end{array} \quad \begin{array}{l} \text{A.index} \\ \text{A.value} \end{array}$$

Figure 11: The data structure of the ELL format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int          n,maxnzc;
2: int          *index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; maxnzc = 3;
6: index = (int *)malloc( n*maxnzc*sizeof(int) );
7: value = (LIS_SCALAR *)malloc( n*maxnzc*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0; index[4] = 0; index[5] = 1;
12: index[6] = 2; index[7] = 2; index[8] = 0; index[9] = 1; index[10] = 2; index[11] = 3;
13: value[0] = 11; value[1] = 21; value[2] = 32; value[3] = 41; value[4] = 0; value[5] = 22;
14: value[6] = 33; value[7] = 43; value[8] = 0; value[9] = 0; value[10] = 0; value[11] = 44;
15:
16: lis_matrix_set_ell(maxnzc,index,value,A);
17: lis_matrix_assemble(A);

```

5.5.2 Creating Matrices (for MPI Environment)

Figure 12 shows how the matrix A in Figure 11 is stored in the ELL format. A program to create the matrix in the ELL format on two processing elements is as follows:

0	0	0	1	1	0	2	2	2	3	A.index
11	21	0	22	32	41	33	43	0	44	A.value
PE0				PE1						

Figure 12: The data structure of the ELL format (for MPI environment).

for MPI environment

```

1: int          i,n,maxnzs,my_rank;
2: int          *index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; maxnzs = 2;}
7: else          {n = 2; maxnzs = 3;}
8: index = (int *)malloc( n*maxnzs*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( n*maxnzs*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:     index[0] = 0; index[1] = 0; index[2] = 0; index[3] = 1;
14:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;}
15: else {
16:     index[0] = 1; index[1] = 0; index[2] = 2; index[3] = 2; index[4] = 2;
17:     index[5] = 3;
18:     value[0] = 32; value[1] = 41; value[2] = 33; value[3] = 43; value[4] = 0;
19:     value[5] = 44;}
20: lis_matrix_set_ell(maxnzs,index,value,A);
21: lis_matrix_assemble(A);

```

5.5.3 Associating Arrays

To associate an array required by the ELL format with the matrix A , the following functions are used:

- C `int lis_matrix_set_ell(int maxnzs, int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_ell(integer maxnzs, integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.6 Jagged Diagonal (JDS)

The JDS format first sorts the nonzero elements of the rows in decreasing order of size, and then stores them along the column. The JDS format uses four arrays **perm**, **ptr**, **index** and **value** to store data. Assume that $maxn_zr$ represents the maximum value of the number of the nonzero elements of the matrix A .

- **perm** is an integer array with a length of n , which stores the sorted row numbers.
- **value** is a double precision array with a length of nnz , which stores the jagged diagonal elements of the sorted matrix A . The first jagged diagonal consists of the first nonzero elements of each row. The next jagged diagonal consists of the second nonzero elements, and so on.
- **index** is an integer array with a length of nnz , which stores the row numbers of the nonzero elements stored in the array **value**.
- **ptr** is an integer array with a length of $maxn_zr + 1$, which stores the starting points of the jagged diagonal elements.

For the multithreaded environment, the following modifications have been made: The JDS format uses four arrays **perm**, **ptr**, **index** and **value** to store data. Assume that $nprocs$ is the number of the threads. $maxn_zr_p$ is the number of the nonzero diagonal elements of the partial matrix into which the row block of the matrix A is divided. $maxmaxn_zr$ is the maximum value of $maxn_zr_p$.

- **perm** is an integer array with a length of n , which stores the sorted row numbers.
- **value** is a double precision array with a length of nnz , which stores jagged diagonal elements of the sorted matrix A . The first jagged diagonal consists of the first nonzero elements of each row. The next jagged diagonal consist of the second nonzero elements of each row, and so on.
- **index** is an integer array with a length of nnz , which stores the row numbers of the nonzero elements stored in the array **value**.
- **ptr** is an integer array with a length of $nprocs \times (maxmaxn_zr + 1)$, which stores the starting points of the jagged diagonal elements.

5.6.1 Creating Matrices (for Serial Environment)

The right diagram in Figure 13 shows how the matrix A in Figure 13 is stored in the JDS format. A program to create the matrix in the JDS format is as follows:

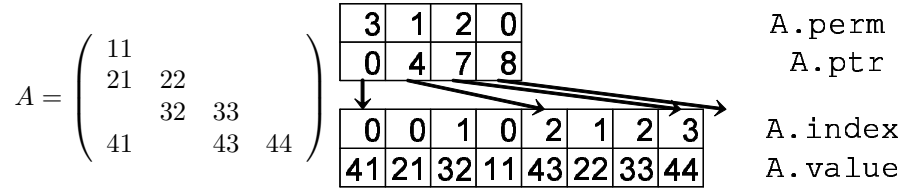


Figure 13: The data structure of the JDS format (for serial environment).

for serial environment

```

1: int      n,nnz,maxnzs;
2: int      *perm,*ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8; maxnzs = 3;
6: perm = (int *)malloc( n*sizeof(int) );
7: ptr = (int *)malloc( (maxnzs+1)*sizeof(int) );
8: index = (int *)malloc( nnz*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
10: lis_matrix_create(0,&A);
11: lis_matrix_set_size(A,0,n);
12:
13: perm[0] = 3; perm[1] = 1; perm[2] = 2; perm[3] = 0;
14: ptr[0] = 0; ptr[1] = 4; ptr[2] = 7; ptr[3] = 8;
15: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0;
16: index[4] = 2; index[5] = 1; index[6] = 2; index[7] = 3;
17: value[0] = 41; value[1] = 21; value[2] = 32; value[3] = 11;
18: value[4] = 43; value[5] = 22; value[6] = 33; value[7] = 44;
19:
20: lis_matrix_set_jds(nnz,maxnzs,perm,ptr,index,value,A);
21: lis_matrix_assemble(A);

```

5.6.2 Creating Matrices (for Multithreaded Environment)

Figure 14 shows how the matrix A in Figure 13 is stored in the JDS format on two threads. A program to create the matrix in the JDS format on two threads is as follows:

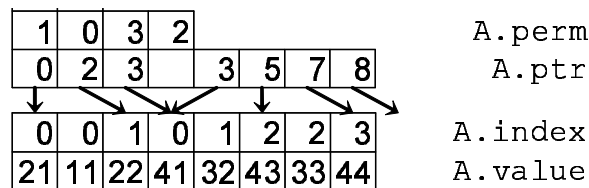


Figure 14: The data structure of the JDS format (for multithreaded environment).

```

for multithreaded environment
1: int      n, nnz, maxmaxnzs, nprocs;
2: int      *perm, *ptr, *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8; maxmaxnzs = 3; nprocs = 2;
6: perm = (int *)malloc( n*sizeof(int) );
7: ptr = (int *)malloc( nprocs*(maxmaxnzs+1)*sizeof(int) );
8: index = (int *)malloc( nnz*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
10: lis_matrix_create(0, &A);
11: lis_matrix_set_size(A, 0, n);
12:
13: perm[0] = 1; perm[1] = 0; perm[2] = 3; perm[3] = 2;
14: ptr[0] = 0; ptr[1] = 2; ptr[2] = 3; ptr[3] = 0;
15: ptr[4] = 3; ptr[5] = 5; ptr[6] = 7; ptr[7] = 8;
16: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0;
17: index[4] = 1; index[5] = 2; index[6] = 2; index[7] = 3;
18: value[0] = 21; value[1] = 11; value[2] = 22; value[3] = 41;
19: value[4] = 32; value[5] = 43; value[6] = 33; value[7] = 44;
20:
21: lis_matrix_set_jds(nnz, maxmaxnzs, perm, ptr, index, value, A);
22: lis_matrix_assemble(A);

```

5.6.3 Creating Matrices (for MPI Environment)

Figure 15 shows how the matrix A in Figure 13 is stored in the JDS format on two processing elements. A program to create the matrix in the JDS format on two processing elements is as follows:

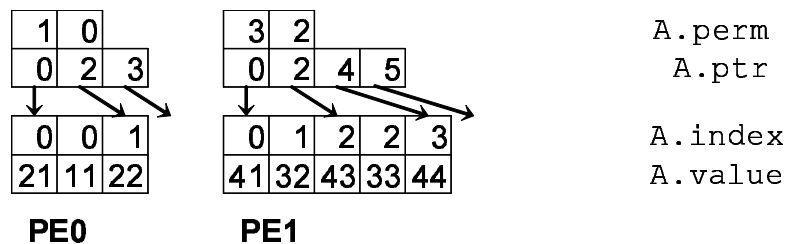


Figure 15: The data structure of the JDS format (for MPI environment).

for MPI environment

```

1: int      i,n,nnz,maxnzs,my_rank;
2: int      *perm,*ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3; maxnzs = 2;}
7: else      {n = 2; nnz = 5; maxnzs = 3;}
8: perm = (int *)malloc( n*sizeof(int) );
9: ptr = (int *)malloc( (maxnzs+1)*sizeof(int) );
10: index = (int *)malloc( nnz*sizeof(int) );
11: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
12: lis_matrix_create(MPI_COMM_WORLD,&A);
13: lis_matrix_set_size(A,n,0);
14: if( my_rank==0 ) {
15:     perm[0] = 1; perm[1] = 0;
16:     ptr[0] = 0; ptr[1] = 2; ptr[2] = 3;
17:     index[0] = 0; index[1] = 0; index[2] = 1;
18:     value[0] = 21; value[1] = 11; value[2] = 22;}
19: else {
20:     perm[0] = 3; perm[1] = 2;
21:     ptr[0] = 0; ptr[1] = 2; ptr[2] = 4; ptr[3] = 5;
22:     index[0] = 0; index[1] = 1; index[2] = 2; index[3] = 2; index[4] = 3;
23:     value[0] = 41; value[1] = 32; value[2] = 43; value[3] = 33; value[4] = 44;}
24: lis_matrix_set_jds(nnz,maxnzs,perm,ptr,index,value,A);
25: lis_matrix_assemble(A);

```

5.6.4 Associating Arrays

To associate an array required by the JDS format with the matrix A , the following functions are used:

- C `int lis_matrix_set_jds(int nnz, int maxnzs, int perm[], int ptr[], int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_jds(integer nnz, integer maxnzs, integer ptr(), integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.7 Block Sparse Row (BSR)

The BSR format breaks down the matrix A into partial matrices called blocks, with a size of $r \times c$. The BSR format stores nonzero blocks, in which at least one nonzero element exists, with the same step as that for the CRS format. Assume that $nr = n/r$ and $nnzb$ are the numbers of nonzero blocks of A . The BSR format uses three arrays **bp**tr, **bin**dex and **val**ue to store matrices.

- **value** is a double precision array with a length of $nnzb \times r \times c$, which stores all the elements of the nonzero blocks.
- **bin**dex is an integer array with a length of $nnzb$, which stores block column numbers of the nonzero blocks.
- **bp**tr is an integer array with a length of $nr + 1$, which stores the starting points of the block rows in the array **bin**dex.

5.7.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 16 shows how the matrix A in Figure 16 is stored in the BSR format. A program to create the matrix in the BSR format is as follows:

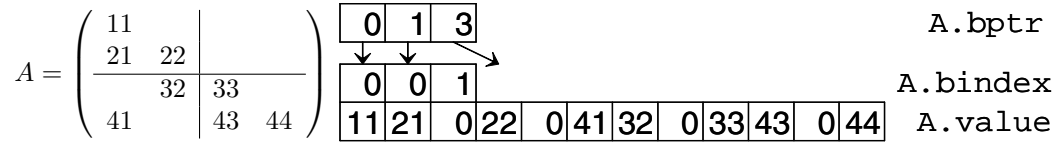


Figure 16: The data structure of the BSR format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int          n, bnr, bnc, nr, nc, bnnz;
2: int          *bp
```

tr, *bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; bnr = 2; bnc = 2; bnnz = 3; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;
6: bptr = (int *)malloc((nr+1)*sizeof(int));
7: bindex = (int *)malloc(bnnz*sizeof(int));
8: value = (LIS_SCALAR *)malloc(bnr*bnc*bnnz*sizeof(LIS_SCALAR));
9: lis_matrix_create(0, &A);
10: lis_matrix_set_size(A, 0, n);
11:
12: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
13: bindex[0] = 0; bindex[1] = 0; bindex[2] = 1;
14: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
15: value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
16: value[8] = 33; value[9] = 43; value[10] = 0; value[11] = 44;
17:
18: lis_matrix_set_bsr(bnr, bnc, bnnz, bptr, bindex, value, A);
19: lis_matrix_assemble(A);

5.7.2 Creating Matrices (for MPI Environment)

Figure 17 shows how the matrix A in Figure 16 is stored in the BSR format on two processing elements. A program to create the matrix in the BSR format on two processing elements is as follows:

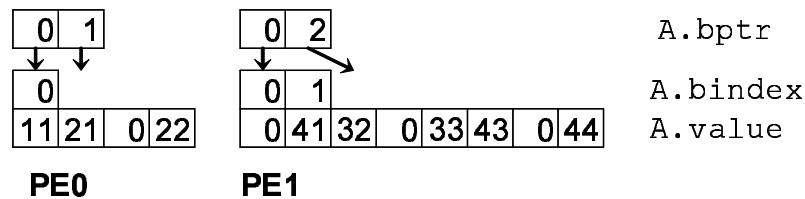


Figure 17: The data structure of the BSR format (for MPI environment).

for MPI environment

```

1: int      n, bnr, bnc, nr, nc, bnnz, my_rank;
2: int      *bptr, *bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
6: if( my_rank==0 ) {n = 2; bnr = 2; bnc = 2; bnnz = 1; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
7: else      {n = 2; bnr = 2; bnc = 2; bnnz = 2; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
8: bptr = (int *)malloc( (nr+1)*sizeof(int) );
9: bindex = (int *)malloc( bnnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD, &A);
12: lis_matrix_set_size(A, n, 0);
13: if( my_rank==0 ) {
14:     bptr[0] = 0; bptr[1] = 1;
15:     bindex[0] = 0;
16:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;}
17: else {
18:     bptr[0] = 0; bptr[1] = 2;
19:     bindex[0] = 0; bindex[1] = 1;
20:     value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 0;
21:     value[4] = 33; value[5] = 43; value[6] = 0; value[7] = 44;}
22: lis_matrix_set_bsr(bnr, bnc, bnnz, bptr, bindex, value, A);
23: lis_matrix_assemble(A);

```

5.7.3 Associating Arrays

To associate the arrays in the BSR format with the matrix A , the following functions are used:

- C `int lis_matrix_set_bsr(int bnr, int bnc, int bnnz, int bptr[], int bindex[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_bsr(integer bnr, integer bnc, integer bnnz, integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.8 Block Sparse Column (BSC)

The BSC format breaks down the matrix A into partial matrices called blocks, with a size of $r \times c$. The BSC format stores nonzero blocks, in which at least one nonzero block exists, in the same step as that for the CCS format. Assume that $nc = n/c$ and $nnzb$ are the numbers of nonzero blocks of A . The BSC format uses three arrays **bptr**, **bindex** and **value** to store matrices.

- **value** is a double precision array with a length of $nnzb \times r \times c$, which stores all the elements of the nonzero blocks.
- **bindex** is an integer array with a length of $nnzb$, which stores the block row numbers of the nonzero blocks.
- **bptr** is an integer array with a length of $nc+1$, which stores the starting points of the block columns in the array **bindex**.

5.8.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 18 shows how the matrix A in Figure 18 is stored in the BSC format. A program to create the matrix in the BSC format is as follows:

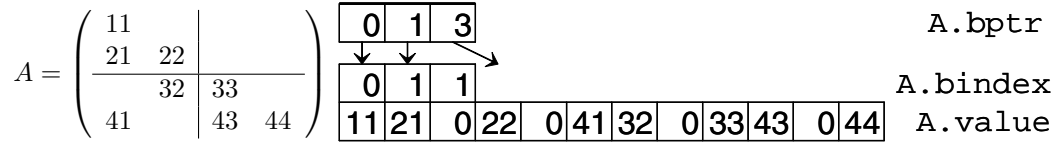


Figure 18: The data structure of the BSC format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int          n, bnr, bnc, nr, nc, bnnz;
2: int          *bptr, *bindex;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; bnr = 2; bnc = 2; bnnz = 3; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;
6: bptr = (int *)malloc( (nc+1)*sizeof(int) );
7: bindex = (int *)malloc( bnnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0, &A);
10: lis_matrix_set_size(A, 0, n);
11:
12: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
13: bindex[0] = 0; bindex[1] = 1; bindex[2] = 1;
14: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
15: value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
16: value[8] = 33; value[9] = 43; value[10] = 0; value[11] = 44;
17:
18: lis_matrix_set_bsc(bnr, bnc, bnnz, bptr, bindex, value, A);
19: lis_matrix_assemble(A);

```

5.8.2 Creating Matrices (for MPI Environment)

Figure 19 shows how the matrix A in Figure 18 is stored in the BSC format on two processing elements. A program to create the matrix in the BSC format on two processing elements is as follows:

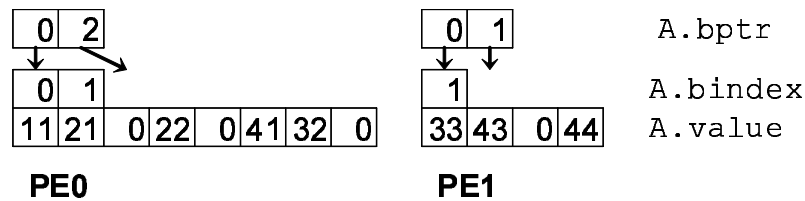


Figure 19: The data structure of the BSC format (for MPI environment).

for MPI environment

```

1: int          n, bnr, bnc, nr, nc, bnnz, my_rank;
2: int          *bptr, *bindex;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
6: if( my_rank==0 ) {n = 2; bnr = 2; bnc = 2; bnnz = 2; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
7: else          {n = 2; bnr = 2; bnc = 2; bnnz = 1; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
8: bptr  = (int *)malloc( (nr+1)*sizeof(int) );
9: bindex = (int *)malloc( bnnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD, &A);
12: lis_matrix_set_size(A, n, 0);
13: if( my_rank==0 ) {
14:     bptr[0] = 0; bptr[1] = 2;
15:     bindex[0] = 0; bindex[1] = 1;
16:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
17:     value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
18: } else {
19:     bptr[0] = 0; bptr[1] = 1;
20:     bindex[0] = 1;
21:     value[0] = 33; value[1] = 43; value[2] = 0; value[3] = 44;
22: lis_matrix_set_bsc(bnr, bnc, bnnz, bptr, bindex, value, A);
23: lis_matrix_assemble(A);

```

5.8.3 Associating Arrays

To associate the arrays in the BSC format with the matrix A , the following functions are used:

- C `int lis_matrix_set_bsc(int bnr, int bnc, int bnnz, int bptr[], int bindex[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_bsc(integer bnr, integer bnc, integer bnnz, integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.9 Variable Block Row (VBR)

The VBR format is the generalized version of the BSR format. The division points of the rows and columns are given by the arrays `row` and `col`. The VBR format stores the nonzero blocks (the blocks in which at least one nonzero block exists) in the same step as that for the CRS format. Assume that nr and nc are the numbers of row and column divisions, respectively, and that $nnzb$ denotes the number of the nonzero blocks of A , and nnz denotes the total number of the elements of the nonzero blocks. The VBR format uses six arrays `bptr`, `bindex`, `row`, `col`, `ptr` and `value` to store matrices.

- `row` is an integer array with a length of $nr + 1$, which stores the starting row number of the block rows.
- `col` is an integer array with a length of $nc + 1$, which stores the starting column number of the block columns.
- `bindex` is an integer array with a length of $nnzb$, which stores the block column numbers of the nonzero blocks.
- `bptr` is an integer array with a length of $nr + 1$, which stores the starting points of the block rows in the array `bindex`.
- `value` is a double precision array with a length of nnz , which stores all the elements of the nonzero blocks.
- `ptr` is an integer array with a length of $nnzb + 1$, which stores the starting points of the nonzero blocks in the array `value`.

5.9.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 20 shows how the matrix A in Figure 20 is stored in the VBR format. A program to create the matrix in the VBR format is as follows:

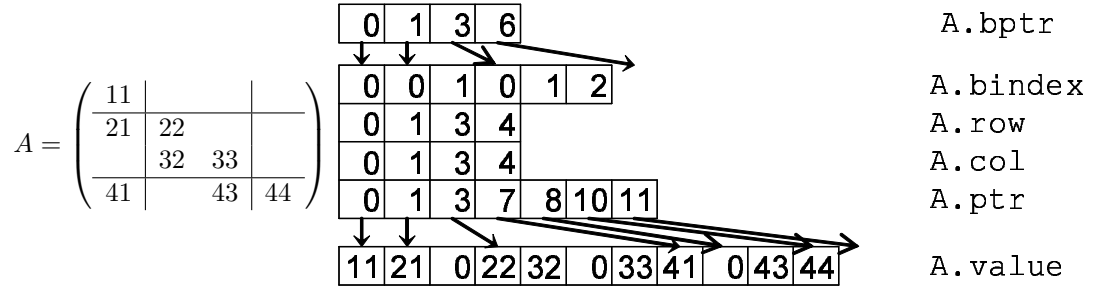


Figure 20: The data structure of the VBR format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int          n,nnz,nr,nc,bnnz;
2: int          *row,*col,*ptr,*bptr,*bindex;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; nnz = 11; bnnz = 6; nr = 3; nc = 3;
6: bptr  = (int *)malloc( (nr+1)*sizeof(int) );
7: row   = (int *)malloc( (nr+1)*sizeof(int) );
8: col   = (int *)malloc( (nc+1)*sizeof(int) );
9: ptr   = (int *)malloc( (bnnz+1)*sizeof(int) );
10: bindex = (int *)malloc( bnnz*sizeof(int) );
11: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
12: lis_matrix_create(0,&A);
13: lis_matrix_set_size(A,0,n);
14:
15: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3; bptr[3] = 6;
16: row[0]  = 0; row[1]  = 1; row[2]  = 3; row[3]  = 4;
17: col[0]  = 0; col[1]  = 1; col[2]  = 3; col[3]  = 4;
18: bindex[0] = 0; bindex[1] = 0; bindex[2] = 1; bindex[3] = 0;
19: bindex[4] = 1; bindex[5] = 2;
20: ptr[0]   = 0; ptr[1]   = 1; ptr[2]   = 3; ptr[3]   = 7;
21: ptr[4]   = 8; ptr[5]   = 10; ptr[6]   = 11;
22: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
23: value[4] = 32; value[5] = 0; value[6] = 33; value[7] = 41;
24: value[8] = 0; value[9] = 43; value[10] = 44;
25:
26: lis_matrix_set_vbr(nnz,nr,nc,bnnz,row,col,ptr,bptr,bindex,value,A);
27: lis_matrix_assemble(A);

```

5.9.2 Creating Matrices (for MPI Environment)

Figure 21 shows how the matrix A in Figure 20 is stored in the VBR format on two processing elements. A program to create the matrix in the VBR format on two processing elements is as follows:

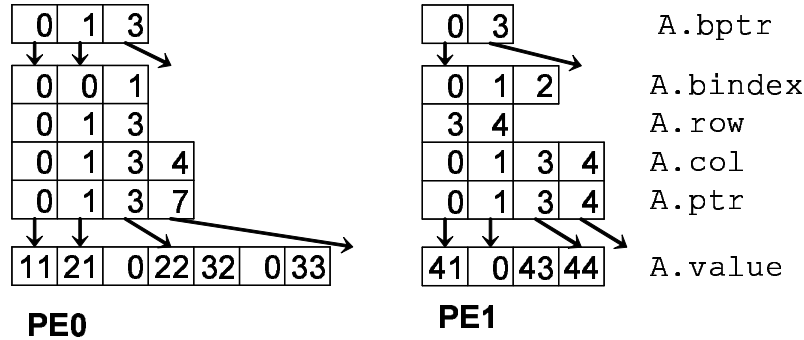


Figure 21: The data structure of the VBR format (for MPI environment).

for MPI environment

```

1: int          n,nnz,nr,nc,bnnz,my_rank;
2: int          *row,*col,*ptr,*bptr,*bindex;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 7; bnnz = 3; nr = 2; nc = 3;}
7: else          {n = 2; nnz = 4; bnnz = 3; nr = 1; nc = 3;}
8: bptr  = (int *)malloc( (nr+1)*sizeof(int) );
9: row   = (int *)malloc( (nr+1)*sizeof(int) );
10: col   = (int *)malloc( (nc+1)*sizeof(int) );
11: ptr   = (int *)malloc( (bnnz+1)*sizeof(int) );
12: bindex = (int *)malloc( bnnz*sizeof(int) );
13: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
14: lis_matrix_create(MPI_COMM_WORLD,&A);
15: lis_matrix_set_size(A,n,0);
16: if( my_rank==0 ) {
17:     bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
18:     row[0]  = 0; row[1]  = 1; row[2]  = 3;
19:     col[0]  = 0; col[1]  = 1; col[2]  = 3; col[3] = 4;
20:     bindex[0] = 0; bindex[1] = 0; bindex[2] = 1;
21:     ptr[0]   = 0; ptr[1]   = 1; ptr[2]   = 3; ptr[3]   = 7;
22:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
23:     value[4] = 32; value[5] = 0; value[6] = 33;}
24: else {
25:     bptr[0] = 0; bptr[1] = 3;
26:     row[0]  = 3; row[1]  = 4;
27:     col[0]  = 0; col[1]  = 1; col[2]  = 3; col[3] = 4;
28:     bindex[0] = 0; bindex[1] = 1; bindex[2] = 2;
29:     ptr[0]   = 0; ptr[1]   = 1; ptr[2]   = 3; ptr[3]   = 4;
30:     value[0] = 41; value[1] = 0; value[2] = 43; value[3] = 44;}
31: lis_matrix_set_vbr(nnz,nr,nc,bnnz,row,col,ptr,bptr,bindex,value,A);
32: lis_matrix_assemble(A);

```

5.9.3 Associating Arrays

To associate the arrays in the VBR format with the matrix A , the following functions are used:

- C `int lis_matrix_set_vbr(int nnz, int nr, int nc, int bnnz, int row[], int col[], int ptr[], int bptr[], int bindex[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_vbr(integer nnz, integer nr, integer nc, integer bnnz, integer row(), integer col(), integer ptr(), integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.10 Coordinate (COO)

The COO format uses three arrays `row`, `col` and `value` to store data.

- `value` is a double precision array with a length of nnz , which stores the nonzero elements.
- `row` is an integer array with a length of nnz , which stores the row numbers of the nonzero elements.
- `col` is an integer array with a length of nnz , which stores the column numbers of the nonzero elements.

5.10.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 22 shows how the matrix A in Figure 22 is stored in the COO format. A program to create the matrix in the COO format is as follows:

$$A = \begin{pmatrix} 11 & & & \\ 21 & 22 & & \\ & 32 & 33 & \\ 41 & & 43 & 44 \end{pmatrix} \quad \begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 1 & 3 & 1 & 2 & 2 & 3 & 3 \\ \hline 0 & 0 & 0 & 1 & 1 & 2 & 2 & 3 \\ \hline 11 & 21 & 41 & 22 & 32 & 33 & 43 & 44 \\ \hline \end{array} \quad \begin{array}{l} A.\text{row} \\ A.\text{col} \\ A.\text{value} \end{array}$$

Figure 22: The data structure of the COO format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int      n,nnz;
2: int      *row,*col;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8;
6: row = (int *)malloc( nnz*sizeof(int) );
7: col = (int *)malloc( nnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: row[0] = 0; row[1] = 1; row[2] = 3; row[3] = 1;
13: row[4] = 2; row[5] = 2; row[6] = 3; row[7] = 3;
14: col[0] = 0; col[1] = 0; col[2] = 0; col[3] = 1;
15: col[4] = 1; col[5] = 2; col[6] = 2; col[7] = 3;
16: value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22;
17: value[4] = 32; value[5] = 33; value[6] = 43; value[7] = 44;
18:
19: lis_matrix_set_coo(nnz,row,col,value,A);
20: lis_matrix_assemble(A);

```


5.10.2 Creating Matrices (for MPI Environment)

Figure 23 shows how the matrix A in Figure 22 is stored in the COO format on two processing elements. A program to create the matrix in the COO format on two processing elements is as follows:

0	1	1	3	2	2	3	3	A.row
0	0	1	0	1	2	2	3	A.col
11	21	22	41	32	33	43	44	A.value
PE0			PE1					

Figure 23: The data structure of the COO format (for MPI environment).

for MPI environment

```

1: int      n,nnz,my_rank;
2: int      *row,*col;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else      {n = 2; nnz = 5;}
8: row  = (int *)malloc( nnz*sizeof(int) );
9: col  = (int *)malloc( nnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:     row[0] = 0; row[1] = 1; row[2] = 1;
15:     col[0] = 0; col[1] = 0; col[2] = 1;
16:     value[0] = 11; value[1] = 21; value[2] = 22;}
17: else {
18:     row[0] = 3; row[1] = 2; row[2] = 2; row[3] = 3; row[4] = 3;
19:     col[0] = 0; col[1] = 1; col[2] = 2; col[3] = 2; col[4] = 3;
20:     value[0] = 41; value[1] = 32; value[2] = 33; value[3] = 43; value[4] = 44;}
21: lis_matrix_set_coo(nnz,row,col,value,A);
22: lis_matrix_assemble(A);

```

5.10.3 Associating Arrays

To associate the arrays in the COO format with the matrix A , the following functions are used:

- C `int lis_matrix_set_coo(int nnz, int row[], int col[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_coo(integer nnz, integer row(), integer col(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

5.11 Dense (DNS)

The DNS format uses one array `value` to store data.

- `value` is a double precision array with a length of $n \times n$, which stores the elements with priority given to the columns.

5.11.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 24 shows how the matrix A in Figure 24 is stored in the DNS format. A program to create the matrix in the DNS format is as follows:

$$A = \begin{pmatrix} 11 & & & \\ 21 & 22 & & \\ & 32 & 33 & \\ 41 & & 43 & 44 \end{pmatrix} \quad \begin{array}{|c|c|c|c|c|c|c|c|} \hline 11 & 21 & 0 & 41 & 0 & 22 & 32 & 0 \\ \hline 0 & 0 & 33 & 43 & 0 & 0 & 0 & 44 \\ \hline \end{array} \quad \text{A.Value}$$

Figure 24: The data structure of the DNS format (for serial and multithreaded environments).

for serial and multithreaded environments

```

1: int          n;
2: LIS_SCALAR   *value;
3: LIS_MATRIX   A;
4: n = 4;
5: value = (LIS_SCALAR *)malloc( n*n*sizeof(LIS_SCALAR) );
6: lis_matrix_create(0,&A);
7: lis_matrix_set_size(A,0,n);
8:
9: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 41;
10: value[4] = 0; value[5] = 22; value[6] = 32; value[7] = 0;
11: value[8] = 0; value[9] = 0; value[10] = 33; value[11] = 43;
12: value[12] = 0; value[13] = 0; value[14] = 0; value[15] = 44;
13:
14: lis_matrix_set_dns(value,A);
15: lis_matrix_assemble(A);

```

5.11.2 Creating Matrices (for MPI Environment)

Figure 25 shows how the matrix A in Figure 24 is stored in the DNS format on two processing elements. A program to create the matrix in the DNS format on two processing elements is as follows:

11	21	0	22	0	41	32	0	A.Value
0	0	0	0	33	43	0	44	
PE0				PE1				

Figure 25: The data structure of the DNS format (for MPI environment).

for MPI environment

```

1: int          n,my_rank;
2: LIS_SCALAR   *value;
3: LIS_MATRIX   A;
4: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
5: if( my_rank==0 ) {n = 2;}
6: else          {n = 2;}
7: value = (LIS_SCALAR *)malloc( n*n*sizeof(LIS_SCALAR) );
8: lis_matrix_create(MPI_COMM_WORLD,&A);
9: lis_matrix_set_size(A,n,0);
10: if( my_rank==0 ) {
11:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
12:     value[4] = 0; value[5] = 0; value[6] = 0; value[7] = 0;}
13: else {
14:     value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 0;
15:     value[4] = 33; value[5] = 43; value[6] = 0; value[7] = 44;}
16: lis_matrix_set_dns(value,A);
17: lis_matrix_assemble(A);

```

5.11.3 Associating Arrays

To associate the arrays in the DNS format with the matrix A , the following functions are used:

- C `int lis_matrix_set_dns(LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_dns(LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

6 Functions

This section describes the functions which can be employed by the user. The return values of the functions in C and the values of `ierr` in Fortran are as follows:

Return Values

<code>LIS_SUCCESS(0)</code>	Normal termination
<code>LIS_ILL_OPTION(1)</code>	Illegal option
<code>LIS_BREAKDOWN(2)</code>	Breakdown
<code>LIS_OUT_OF_MEMORY(3)</code>	Insufficient working memory
<code>LIS_MAXITER(4)</code>	Did not converge within the maximum number of iterations
<code>LIS_NOT_IMPLEMENTED(5)</code>	Not implemented
<code>LIS_ERR_FILE_IO(6)</code>	File I/O error

6.1 Operating Vector Elements

Assume that the size of the vector v is $global_n$ and that the size of the partial vectors stored on $nprocs$ processing elements is $local_n$. $global_n$ and $local_n$ are called the global size and the local size, respectively.

6.1.1 `lis_vector_create`

```
C      int lis_vector_create(LIS_Comm comm, LIS_VECTOR *v)
Fortran subroutine lis_vector_create(LIS_Comm comm, LIS_VECTOR v, integer ierr)
```

Description

Create the vector v

Input

<code>LIS_Comm</code>	MPI communicator
-----------------------	------------------

Output

<code>v</code>	Vector
<code>ierr</code>	Return code

Note

For the serial and multithreaded environments, the value for `comm` is ignored.

6.1.2 lis_vector_destroy

```
C      int lis_vector_destroy(LIS_VECTOR v)
Fortran subroutine lis_vector_destroy(LIS_VECTOR v, integer ierr)
```

Description

Destroy the vector v

Input

<code>v</code>	Vector to be destroyed
----------------	------------------------

Output

<code>ierr</code>	Return code
-------------------	-------------

6.1.3 lis_vector_duplicate

```
C      int lis_vector_duplicate(void *vin, LIS_VECTOR *vout)
Fortran subroutine lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR vout,
      integer ierr)
```

Description

Create the vector v_{out} which has the same information as v_{in}

Input

<code>vin</code>	Source vector
------------------	---------------

Output

<code>vout</code>	Destination vector
<code>ierr</code>	Return code

Note

The function `lis_vector_duplicate` does not copy the values, but only allocates memory. To copy the values as well, the function `lis_vector_copy` must be called after this function.

6.1.4 lis_vector_set_size

```
C      int lis_vector_set_size(LIS_VECTOR v, int local_n, int global_n)
Fortran subroutine lis_vector_set_size(LIS_VECTOR v, integer local_n,
      integer global_n, integer ierr)
```

Description

Assign the size of the vector v

Input

<code>v</code>	Vector
<code>local_n</code>	Size of partial vector
<code>global_n</code>	Size of global vector

Output

<code>ierr</code>	Return code
-------------------	-------------

Note

Either *local_n* or *global_n* must be provided.

In the case of the serial and multithreaded environments, *local_n* is equal to *global_n*. Therefore, both `lis_vector_set_size(v,n,0)` and `lis_vector_set_size(v,0,n)` create a vector of size n .

For the MPI environment, `lis_vector_set_size(v,n,0)` creates a partial vector of size n on each processing element. On the other hand, `lis_vector_set_size(v,0,n)` creates a partial vector of size m_p on the processing element p . The values of m_p are determined by the library.

6.1.5 lis_vector_get_size

```
C      int lis_vector_get_size(LIS_VECTOR v, int *local_n, int *global_n)
Fortran subroutine lis_vector_get_size(LIS_VECTOR v, integer local_n,
      integer global_n, integer ierr)
```

Description

Get the size of the vector v

Input

<code>v</code>	Vector
----------------	--------

Output

<code>local_n</code>	Size of partial vector
<code>global_n</code>	Size of global vector
<code>ierr</code>	Return code

Note

In the case of the serial and multithreaded environments, *local_n* is equal to *global_n*.

6.1.6 lis_vector_get_range

```
C      int lis_vector_get_range(LIS_VECTOR v, int *is, int *ie)
Fortran subroutine lis_vector_get_range(LIS_VECTOR v, integer is, integer ie,
      integer ierr)
```

Description

Get the location of the partial vector v in the global vector

Input

<code>v</code>	Partial vector
----------------	----------------

Output

<code>is</code>	Location where partial vector v starts in global vector
<code>ie</code>	1+ location where partial vector v ends in global vector
<code>ierr</code>	Return code

Note

For the serial and multithreaded environments, a vector of size n results in $is = 0$ and $ie = n$.

6.1.7 lis_vector_set_value

```
C      int lis_vector_set_value(int flag, int i, LIS_SCALAR value, LIS_VECTOR v)
Fortran subroutine lis_vector_set_value(integer flag, integer i, LIS_SCALAR value,
      LIS_VECTOR v, integer ierr)
```

Description

Assign the scalar $value$ to the i -th row of the vector v

Input

<code>flag</code>	<code>LIS_INS_VALUE</code> : $v[i] = value$ <code>LIS_ADD_VALUE</code> : $v[i] = v[i] + value$
<code>i</code>	Location where value is assigned
<code>value</code>	Scalar value to be assigned
<code>v</code>	Destination vector

Output

<code>v</code>	Vector with scalar $value$ assigned to the i -th row
<code>ierr</code>	Return code

Note

For the MPI environment, the i -th row of the global vector must be specified instead of the i -th row of the partial vector.

6.1.8 lis_vector_get_value

```
C      int lis_vector_get_value(LIS_VECTOR v, int i, LIS_SCALAR *value)
Fortran subroutine lis_vector_get_value(LIS_VECTOR v, integer i, LIS_SCALAR value,
      integer ierr)
```

Description

Get the value of the i -th row of the vector v

Input

i	Location where value should be assigned
v	Destination vector

Output

$value$	Value of the i -th row
$ierr$	Return code

Note

For the MPI environment, the i -th row of the global vector must be specified instead of the i -th row of the partial vector.

6.1.9 lis_vector_set_values

```
C      int lis_vector_set_values(int flag, int count, int index[],
      LIS_SCALAR value[], LIS_VECTOR v)
Fortran subroutine lis_vector_set_values(integer flag, integer count,
      integer index(), LIS_SCALAR value(), LIS_VECTOR v, integer ierr)
```

Description

Assign the scalar values $value[i]$ to the $index[i]$ -th rows of the vector v

Input

$flag$	LIS_INS_VALUE : $v[index[i]] = value[i]$ LIS_ADD_VALUE : $v[index[i]] = v[index[i]] + value[i]$
$count$	Number of elements of array which stores scalar values to be assigned
$index$	Array which stores location where scalar values should be assigned
$value$	Array which stores scalar values to be assigned
v	Destination vector

Output

v	Vector with scalar $value[i]$ assigned to the $index[i]$ -th row
$ierr$	Return code

Note

For the MPI environment, the $index[i]$ -th row of the global vector must be specified instead of the $index[i]$ -th row of the partial vector.

6.1.10 lis_vector_get_values

```
C      int lis_vector_get_values(LIS_VECTOR v, int start, int count,  
                               LIS_SCALAR value[])  
Fortran subroutine lis_vector_get_values(LIS_VECTOR v, integer start,  
                                         integer count, LIS_SCALAR value(), integer ierr)
```

Description

Get the scalar values of the $start + i$ -th row of the vector v , where $i = 0, 1, \dots, count - 1$

Input

<code>start</code>	Starting location
<code>count</code>	Number of values to get
<code>v</code>	Destination vector

Output

<code>value</code>	Vector to store scalar values
<code>ierr</code>	Return code

Note

For the MPI environment, the $start + i$ -th row of the global vector must be specified instead of the $start + i$ -th row of the partial vector.

6.1.11 lis_vector_scatter

```
C      int lis_vector_scatter(LIS_SCALAR value[], LIS_VECTOR v)
Fortran subroutine lis_vector_scatter(LIS_SCALAR value(), LIS_VECTOR v, integer ierr)
```

Description

Assign the scalar values of the i -th row of the vector v , where $i = 0, 1, \dots, global_n - 1$

Input

value	Array which stores scalar values to be assigned
-------	---

Output

v	Destination vector
ierr	Return code

Note

6.1.12 lis_vector_gather

```
C      int lis_vector_gather(LIS_VECTOR v, LIS_SCALAR value[])
Fortran subroutine lis_vector_gather(LIS_VECTOR v, LIS_SCALAR value(), integer ierr)
```

Description

Get the scalar values of the i -th row of the vector v , where $i = 0, 1, \dots, global_n - 1$

Input

v	Source vector
---	---------------

Output

value	Vector to store scalar values
ierr	Return code

Note

6.1.13 lis_vector_copy

```
C      int lis_vector_copy(LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_copy(LIS_VECTOR x, LIS_VECTOR y, integer ierr)
```

Description

Copy the vector elements

Input

x	Source vector
---	---------------

Output

y	Destination vector
ierr	Return code

6.1.14 lis_vector_set_all

```
C      int lis_vector_set_all(LIS_SCALAR value, LIS_VECTOR x)
Fortran subroutine lis_vector_set_all(LIS_SCALAR value, LIS_VECTOR x, integer ierr)
```

Description

Assign the scalar *value* to the all elements of the vector *v*

Input

value	Scalar value to be assigned
v	Destination vector

Output

v	Vector with <i>value</i> assigned to all elements
ierr	Return code

6.2 Operating Matrix Elements

Assume that the size of the matrix A is $global_n \times global_n$ and that the size of each partial matrix stored on $nprocs$ processing elements is $local_n \times global_n$. Here, $global_n$ and $local_n$ are called the number of the rows of the global matrix and the number of the rows of the partial matrix, respectively.

6.2.1 lis_matrix_create

```
C      int lis_matrix_create(LIS_Comm comm, LIS_MATRIX *A)
Fortran subroutine lis_matrix_create(LIS_Comm comm, LIS_MATRIX A, integer ierr)
```

Description

Create the matrix A

Input

LIS_Comm	MPI communicator
----------	------------------

Output

A	Matrix
ierr	Return code

Note

For the sequential and the multithreaded environments, the value for `comm` is ignored.

6.2.2 lis_matrix_destroy

```
C      int lis_matrix_destroy(LIS_MATRIX A)
Fortran subroutine lis_matrix_destroy(LIS_MATRIX A, integer ierr)
```

Description

Destroy the matrix A

Input

A	Matrix to be destroyed
---	------------------------

Output

ierr	Return code
------	-------------

6.2.3 lis_matrix_duplicate

```
C      int lis_matrix_duplicate(LIS_MATRIX Ain, LIS_MATRIX *Aout)
Fortran subroutine lis_matrix_duplicate(LIS_MATRIX Ain, LIS_MATRIX Aout,
      integer ierr)
```

Description

Create the matrix A_{out} which has the same information as the original A_{in}

Input

Ain	Source matrix
-----	---------------

Output

Aout	Destination matrix
ierr	Return code

Note

The function `lis_matrix_duplicate` does not copy the values of the elements of the matrix, but only allocates memory. To copy the values of the elements as well, the function `lis_matrix_copy` must be called after this function.

6.2.4 lis_matrix_malloc

```
C      int lis_matrix_malloc(LIS_MATRIX A, int nnz_row, int nnz[])
Fortran subroutine lis_matrix_malloc(LIS_MATRIX A, integer nnz_row, integer nnz[],
      integer ierr)
```

Description

Allocate memory for the matrix A

Input

A	Matrix
nnz_row	Average number of nonzero elements
nnz	Array of numbers of nonzero elements in each row

Output

ierr	Return code
------	-------------

Note

Either `nnz_row` or `nnz` must be provided.

6.2.5 lis_matrix_set_value

```
C      int lis_matrix_set_value(int flag, int i, int j, LIS_SCALAR value,  
                             LIS_MATRIX A)  
Fortran subroutine lis_matrix_set_value(integer flag, integer i, integer j,  
                                       LIS_SCALAR value, LIS_MATRIX A, integer ierr)
```

Description

Assign the scalar *value* to the (i, j) -th element of the matrix *A*

Input

flag	LIS_INS.VALUE : $A(i, j) = value$ LIS_ADD.VALUE : $A(i, j) = A(i, j) + value$
i	Row number of matrix
j	Column number of matrix
value	Value to be assigned
A	Matrix

Output

A	Matrix
ierr	Return code

Note

For the MPI environment, the *i*-th row and the *j*-th column of the global matrix must be specified, instead of the *i*-th row and the *j*-th column of the partial matrix.

The function `lis_matrix_set_value` stores the assigned value in a temporary internal format. For this reason, when `lis_matrix_set_value` is used, the function `lis_matrix_assemble` must be called.

6.2.6 lis_matrix_assemble

```
C      int lis_matrix_assemble(LIS_MATRIX A)  
Fortran subroutine lis_matrix_assemble(LIS_MATRIX A, integer ierr)
```

Description

Assemble the matrix *A* into the specified storage format

Input

A	Matrix
---	--------

Output

A	Matrix assembled into specified storage format
ierr	Return code

6.2.7 lis_matrix_set_size

```
int lis_matrix_set_size(LIS_MATRIX A, int local_n, int global_n)
Fortran subroutine lis_matrix_set_size(LIS_MATRIX A, integer local_n,
integer global_n, integer ierr)
```

Description

Assign the size of the matrix A

Input

A	Matrix
$local_n$	Number of rows of partial matrix
$global_n$	Number of rows of global matrix

Output

$ierr$	Return code
--------	-------------

Note

Either $local_n$ or $global_n$ must be provided.

In the case of the serial and multithreaded environments, $local_n = global_n$. Therefore, both `lis_matrix_set_size(A,n,0)` and `lis_matrix_set_size(A,0,n)` create a matrix of size $n \times n$.

For the MPI environment, `lis_matrix_set_size(A,n,0)` creates a partial matrix of size $n \times N$ on each processing element, where N is the total sum of n . On the other hand, `lis_matrix_set_size(A,0,n)` creates a partial matrix of size $m_p \times n$ on the processing element p . The values of m_p are determined by the library.

6.2.8 lis_matrix_get_size

```
C      int lis_matrix_get_size(LIS_MATRIX A, int *local_n, int *global_n)
Fortran subroutine lis_matrix_get_size(LIS_MATRIX A, integer local_n,
integer global_n, integer ierr)
```

Description

Get the size of the matrix A

Input

A	Matrix
-----	--------

Output

$local_n$	Number of rows of partial matrix
$global_n$	Number of rows of global matrix
$ierr$	Return code

Note

In case of the serial and multithreaded environments, $local_n$ is equal to $global_n$.

6.2.9 lis_matrix_get_range

```
C      int lis_matrix_get_range(LIS_MATRIX A, int *is, int *ie)
Fortran subroutine lis_matrix_get_range(LIS_MATRIX A, integer is, integer ie,
      integer ierr)
```

Description

Get the location of the partial matrix A in the global matrix

Input

A	Partial matrix
---	----------------

Output

is	Location where partial matrix A starts in global matrix
ie	1+ location where partial matrix A ends in global matrix
ierr	Return code

Note

For the serial and multithreaded environments, a matrix of $n \times n$ results in $is = 0$ and $ie = n$.

6.2.10 lis_matrix_set_type

```
C      int lis_matrix_set_type(LIS_MATRIX A, int matrix_type)
Fortran subroutine lis_matrix_set_type(LIS_MATRIX A, int matrix_type, integer ierr)
```

Description

Assign the storage format

Input

A	Matrix
matrix_type	Storage format

Output

ierr	Return code
------	-------------

Note

matrix_type of *A* is LIS_MATRIX_CRS when the matrix is created. The table below shows the available storage formats for matrix_type.

storage format		matrix_type
Compressed Row Storage	(CRS)	LIS_MATRIX_CRS
Compressed Column Storage	(CCS)	LIS_MATRIX_CCS
Modified Compressed Sparse Row	(MSR)	LIS_MATRIX_MSR
Diagonal	(DIA)	LIS_MATRIX_DIA
Ellpack-Itpack generalized diagonal	(ELL)	LIS_MATRIX_ELL
Jagged Diagonal	(JDS)	LIS_MATRIX_JDS
Block Sparse Row	(BSR)	LIS_MATRIX_BSR
Block Sparse Column	(BSC)	LIS_MATRIX_BSC
Variable Block Row	(VBR)	LIS_MATRIX_VBR
Dense	(DNS)	LIS_MATRIX_DNS
Coordinate	(COO)	LIS_MATRIX_COO

6.2.11 lis_matrix_get_type

```
C      int lis_matrix_get_type(LIS_MATRIX A, int *matrix_type)
Fortran subroutine lis_matrix_get_type(LIS_MATRIX A, integer matrix_type,
      integer ierr)
```

Description

Get the storage format

Input

A	Matrix
---	--------

Output

matrix_type	Storage format
ierr	Return code

6.2.12 lis_matrix_set_blocksize

```
C      int lis_matrix_set_blocksize(LIS_MATRIX A, int bnr, int bnc, int row[],
                                   int col[])
Fortran subroutine lis_matrix_set_blocksize(LIS_MATRIX A, integer bnr, integer bnc,
                                           integer row[], integer col[], integer ierr)
```

Description

Assign the block size for the BSR, BSC, and VBR

Input

A	Matrix
bnr	Row block size for BSR (BSC) or number of row blocks for VBR
bnc	Column block size for BSR (BSC) or number of column blocks for VBR
row	Array of row division information about VBR
col	Array of column division information about VBR

Output

ierr	Return code
------	-------------

6.2.13 lis_matrix_convert

```
C      int lis_matrix_convert(LIS_MATRIX Ain, LIS_MATRIX Aout)
Fortran subroutine lis_matrix_convert(LIS_MATRIX Ain, LIS_MATRIX Aout, integer ierr)
```

Description

Convert the matrix A_{in} into A_{out} of the type specified by `lis_matrix_set_type`

Input

Ain	Source matrix
-----	---------------

Output

Aout	Matrix converted into specified type
ierr	Return code

Note

The specification of the converted storage format is set to A_{out} by using `lis_matrix_set_type`. The specification of the block size of BSR, BSC, and VBR is set to A_{out} by using `lis_matrix_set_blocksize`.

In converting the storage type of the source matrix into a specified type, the conversions indicated by one in the table below are performed directly, and the other conversions are made via the indicated types. The conversions with no indication are made via the CRS type.

Src \ Dst	CRS	CCS	MSR	DIA	ELL	JDS	BSR	BSC	VBR	DNS	COO
CRS	1	1	1	1	1	1	1	CCS	1	1	1
COO	1	1	1	CRS	CRS	CRS	CRS	CCS	CRS	CRS	1

6.2.14 lis_matrix_copy

```
C      int lis_matrix_copy(LIS_MATRIX Ain, LIS_MATRIX Aout)
Fortran subroutine lis_matrix_copy(LIS_MATRIX Ain, LIS_MATRIX Aout, integer ierr)
```

Description

Copy the matrix elements

Input

Ain	Source matrix
-----	---------------

Output

Aout	Destination matrix
ierr	Return code

6.2.15 lis_matrix_get_diagonal

```
C      int lis_matrix_get_diagonal(LIS_MATRIX A, LIS_VECTOR d)
Fortran subroutine lis_matrix_get_diagonal(LIS_MATRIX A, LIS_VECTOR d, integer ierr)
```

Description

Store the diagonal elements of the matrix A to the vector d

Input

A	Matrix
---	--------

Output

d	Vector which stores diagonal elements of matrix
ierr	Return code

6.2.16 lis_matrix_set_crs

```
C      int lis_matrix_set_crs(int nnz, int ptr[], int index[], LIS_SCALAR value[],  
                             LIS_MATRIX A)  
Fortran subroutine lis_matrix_set_crs(integer nnz, integer row(), integer index(),  
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the arrays in the CRS format with the matrix A

Input

<code>nnz</code>	Number of nonzero elements
<code>ptr, index, value</code>	Arrays in CRS format
<code>A</code>	Matrix

Output

<code>A</code>	Matrix associated with arrays
----------------	-------------------------------

Note

After `lis_matrix_set_crs` is used, the function `lis_matrix_assemble` must be called.

6.2.17 lis_matrix_set_ccs

```
C      int lis_matrix_set_ccs(int nnz, int ptr[], int index[], LIS_SCALAR value[],  
                             LIS_MATRIX A)  
Fortran subroutine lis_matrix_set_ccs(integer nnz, integer row(), integer index(),  
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the arrays in the CCS format with the matrix A

Input

<code>nnz</code>	Number of nonzero elements
<code>ptr, index, value</code>	Arrays in CCS format
<code>A</code>	Matrix

Output

<code>A</code>	Matrix associated with arrays
----------------	-------------------------------

Note

After `lis_matrix_set_ccs` is used, the function `lis_matrix_assemble` must be called.

6.2.18 lis_matrix_set_msr

```
C      int lis_matrix_set_msr(int nnz, int ndz, int index[], LIS_SCALAR value[],  
                             LIS_MATRIX A)  
Fortran subroutine lis_matrix_set_msr(integer nnz, integer ndz, integer index(),  
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the arrays in the MSR format with the matrix A

Input

nnz	Number of nonzero elements
ndz	Number of nonzero elements in diagonal
index, value	Arrays in MSR format
A	Matrix

Output

A	Matrix associated with arrays
---	-------------------------------

Note

After `lis_matrix_set_msr` is used, the function `lis_matrix_assemble` must be called.

6.2.19 lis_matrix_set_dia

```
C      int lis_matrix_set_dia(int nnd, int index[], LIS_SCALAR value[],  
                             LIS_MATRIX A)  
Fortran subroutine lis_matrix_set_dia(integer nnd, integer index(),  
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the arrays in the DIA format with the matrix A

Input

nnd	Number of nonzero diagonal elements
index, value	Arrays in DIA format
A	Matrix

Output

A	Matrix associated with arrays
---	-------------------------------

Note

After `lis_matrix_set_dia` is used, the function `lis_matrix_assemble` must be called.

6.2.20 lis_matrix_set_ell

```
C      int lis_matrix_set_ell(int maxnzs, int index[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_ell(integer maxnzs, integer index(),
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the arrays in the ELL format with the matrix A

Input

maxnzs	Maximum number of nonzero elements in each row
index, value	Arrays in ELL format
A	Matrix

Output

A	Matrix associated with arrays
---	-------------------------------

Note

After `lis_matrix_set_ell` is used, the function `lis_matrix_assemble` must be called.

6.2.21 lis_matrix_set_jds

```
C      int lis_matrix_set_jds(int nnz, int maxnzs, int perm[], int ptr[],
                             int index[], LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_jds(integer nnz, integer maxnzs, integer ptr(),
                                     integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the arrays in the JDS format with the matrix A

Input

nnz	Number of nonzero elements
maxnzs	Maximum number of nonzero elements in each row
perm, ptr, index, value	Arrays in JDS format
A	Matrix

Output

A	Matrix associated with arrays
---	-------------------------------

Note

After `lis_matrix_set_jds` is used, the function `lis_matrix_assemble` must be called.

6.2.22 lis_matrix_set_bsr

```
C      int lis_matrix_set_bsr(int bnr, int bnc, int bnnz, int bptr[], int bindex[],
                             LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_bsr(integer bnr, integer bnc, integer bnnz,
                                     integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A,
                                     integer ierr)
```

Description

Associate the arrays in the BSR format with the matrix A

Input

bnr	Row block size
bnc	Column block size
bnnz	Number of nonzero blocks
bptr, bindex, value	Arrays in BSR format
A	Matrix

Output

A	Matrix associated with arrays
---	-------------------------------

Note

After `lis_matrix_set_bsr` is used, the function `lis_matrix_assemble` must be called.

6.2.23 lis_matrix_set_bsc

```
C      int lis_matrix_set_bsc(int bnr, int bnc, int bnnz, int bptr[], int bindex[],
                             LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_bsc(integer bnr, integer bnc, integer bnnz,
                                     integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A,
                                     integer ierr)
```

Description

Associate the arrays in the BSC format with the matrix A

Input

bnr	Row block size
bnc	Column block size
bnnz	Number of nonzero blocks
bptr, bindex, value	Arrays in BSC format
A	Matrix

Output

A	Matrix associated with arrays
---	-------------------------------

Note

After `lis_matrix_set_bsc` is used, the function `lis_matrix_assemble` must be called.

6.2.24 lis_matrix_set_vbr

```
C      int lis_matrix_set_vbr(int nnz, int nr, int nc, int bnnz, int row[],
                             int col[], int ptr[], int bptr[], int bindex[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_vbr(integer nnz, integer nr, integer nc,
                                     integer bnnz, integer row(), integer col(), integer ptr(), integer bptr(),
                                     integer bindex(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the arrays in the VBR format with the matrix A

Input

<code>nnz</code>	Number of all nonzero elements
<code>nr</code>	Number of row blocks
<code>nc</code>	Number of column blocks
<code>bnnz</code>	Number of nonzero blocks
<code>row, col, ptr, bptr, bindex, value</code>	Arrays in VBR format
<code>A</code>	Matrix

Output

<code>A</code>	Matrix associated with arrays
----------------	-------------------------------

Note

After `lis_matrix_set_vbr` is used, the function `lis_matrix_assemble` must be called.

6.2.25 lis_matrix_set_coo

```
C      int lis_matrix_set_coo(int nnz, int row[], int col[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_coo(integer nnz, integer row(), integer col(),
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the arrays in the COO format with the matrix A

Input

<code>nnz</code>	Number of nonzero elements
<code>row, col, value</code>	Arrays in COO format
<code>A</code>	Matrix

Output

<code>A</code>	Matrix associated with arrays
----------------	-------------------------------

Note

After `lis_matrix_set_coo` is used, the function `lis_matrix_assemble` must be called.

6.2.26 lis_matrix_set_dns

```
C      int lis_matrix_set_dns(LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_dns(LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

Description

Associate the array in the DNS format with the matrix A

Input

value	Array in DNS format
A	Matrix

Output

A	Matrix associated with array
---	------------------------------

Note

After `lis_matrix_set_dns` is used, the function `lis_matrix_assemble` must be called.

6.3 Operating Vectors and Matrices

6.3.1 lis_vector_scale

```
C      int lis_vector_scale(LIS_SCALAR alpha, LIS_VECTOR x)
Fortran subroutine lis_vector_scale(LIS_SCALAR alpha, LIS_VECTOR x, integer ierr)
```

Description

Multiply the vector x by the scalar α

Input

alpha	Scalar value α
x	Vector to be multiplied

Output

x	Vector multiplied by α
ierr	Return code

6.3.2 lis_vector_dot

```
C      int lis_vector_dot(LIS_VECTOR x, LIS_VECTOR y, LIS_SCALAR *val)
Fortran subroutine lis_vector_dot(LIS_VECTOR x, LIS_VECTOR y, LIS_SCALAR val,
                                integer ierr)
```

Description

Calculate the inner product $x^T y$

Input

x	Vector
y	Vector

Output

val	Inner product value
ierr	Return code

6.3.3 lis_vector_nrm1

```
C      int lis_vector_nrm1(LIS_VECTOR x, LIS_SCALAR *val)
Fortran subroutine lis_vector_nrm1(LIS_VECTOR x, LIS_SCALAR val, integer ierr)
```

Description

Calculate the 1-norm of vector x

Input

<code>x</code>	Vector
----------------	--------

Output

<code>val</code>	1-norm of vector
<code>ierr</code>	Return code

6.3.4 lis_vector_nrm2

```
C      int lis_vector_nrm2(LIS_VECTOR x, LIS_SCALAR *val)
Fortran subroutine lis_vector_nrm2(LIS_VECTOR x, LIS_SCALAR val, integer ierr)
```

Description

Calculate the 2-norm of the vector x

Input

<code>x</code>	Vector
----------------	--------

Output

<code>val</code>	2-norm of vector
<code>ierr</code>	Return code

6.3.5 lis_vector_nrmi

```
C      int lis_vector_nrmi(LIS_VECTOR x, LIS_SCALAR *val)
Fortran subroutine lis_vector_nrmi(LIS_VECTOR x, LIS_SCALAR val, integer ierr)
```

Description

Calculate the infinity norm of the vector x

Input

<code>x</code>	Vector
----------------	--------

Output

<code>val</code>	infinity norm of vector
<code>ierr</code>	Return code

6.3.6 lis_vector_axpy

```
C      int lis_vector_axpy(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_axpy(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y,
                                   integer ierr)
```

Description

Calculate the vector sum $y = \alpha x + y$

Input

alpha	Scalar value
x, y	Vectors

Output

y	$\alpha x + y$ (vector y is overwritten)
ierr	Return code

6.3.7 lis_vector_xpay

```
C      int lis_vector_xpay(LIS_VECTOR x, LIS_SCALAR alpha, LIS_VECTOR y)
Fortran subroutine lis_vector_xpay(LIS_VECTOR x, LIS_SCALAR alpha, LIS_VECTOR y,
                                   integer ierr)
```

Description

Calculate the vector sum $y = x + \alpha y$

Input

alpha	Scalar value
x, y	Vectors

Output

y	$x + \alpha y$ (vector y is overwritten)
ierr	Return code

6.3.8 lis_vector_axpyz

```
C      int lis_vector_axpyz(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y,  
                           LIS_VECTOR z)  
Fortran subroutine lis_vector_axpyz(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y,  
                                   LIS_VECTOR z, integer ierr)
```

Description

Calculate the vector sum $z = \alpha x + y$

Input

alpha	Scalar value
x, y	Vectors

Output

z	$x + \alpha y$
ierr	Return code

6.3.9 lis_matrix_scaling

```
C      int lis_matrix_scaling(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR d, int action)  
Fortran subroutine lis_matrix_scaling(LIS_MATRIX A, LIS_VECTOR b,  
                                   LIS_VECTOR d, integer action, integer ierr)
```

Description

Scale the matrix A

Input

A	Matrix
b	Vector
action	LIS_SCALE_JACOBI : Jacobi scaling $D^{-1}Ax = D^{-1}b$, where D represents the diagonal of $A = (a_{ij})$ LIS_SCALE_SYMM_DIAG : Diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$, where $D^{-1/2}$ represents a diagonal matrix with $1/\sqrt{a_{ii}}$ as diagonal

Output

d	Vector which stores diagonal elements of D^{-1} or $D^{-1/2}$
ierr	Return code

6.3.10 lis_matvec

```
C      void lis_matvec(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_matvec(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
```

Description

Calculate the matrix vector product $y = Ax$

Input

A	Matrix
x	Vector

Output

y	Ax
---	------

6.3.11 lis_matvect

```
C      void lis_matvect(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_matvect(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
```

Description

Calculate the transposed matrix vector product $y = A^T x$

Input

A	Matrix
x	Vector

Output

y	$A^T x$
---	---------

6.4 Solving Linear Equations

6.4.1 lis_solver_create

```
C      int lis_solver_create(LIS_SOLVER *solver)
Fortran subroutine lis_solver_create(LIS_SOLVER solver, integer ierr)
```

Description

Create the solver

Input

None

Output

<code>solver</code>	Solver
<code>ierr</code>	Return code

Note

`solver` has the information on the solver, the preconditioner, etc.

6.4.2 lis_solver_destroy

```
C      int lis_solver_destroy(LIS_SOLVER solver)
Fortran subroutine lis_solver_destroy(LIS_SOLVER solver, integer ierr)
```

Description

Destroy the solver

Input

<code>solver</code>	Solver to be destroyed
---------------------	------------------------

Output

<code>ierr</code>	Return code
-------------------	-------------

6.4.3 lis_solver_set_option

```
C      int lis_solver_set_option(char *text, LIS_SOLVER solver)
Fortran subroutine lis_solver_set_option(character text, LIS_SOLVER solver,
      integer ierr)
```

Description

Set the options for the solver

Input

text Command line options

Output

solver Solver

ierr Return code

Note

The table below shows the available command line options, where `-i {cg|1}` means `-i cg` or `-i 1` and `-maxiter [1000]` indicates that `-maxiter` defaults to 1,000.

Specifying Linear Solvers (Default: `-i bicg`)

Method	Option	Auxiliary Options	
CG	<code>-i {cg 1}</code>		
BiCG	<code>-i {bicg 2}</code>		
CGS	<code>-i {cgs 3}</code>		
BiCGSTAB	<code>-i {bicgstab 4}</code>		
BiCGSTAB(l)	<code>-i {bicgstabl 5}</code>	<code>-ell [2]</code>	Degree l
GPBiCG	<code>-i {gpbicg 6}</code>		
TFQMR	<code>-i {tfqmr 7}</code>		
Orthomin(m)	<code>-i {orthomin 8}</code>	<code>-restart [40]</code>	Restart value m
GMRES(m)	<code>-i {gmres 9}</code>	<code>-restart [40]</code>	Restart value m
Jacobi	<code>-i {jacobi 10}</code>		
Gauss-Seidel	<code>-i {gs 11}</code>		
SOR	<code>-i {sor 12}</code>	<code>-omega [1.9]</code>	Relaxation coefficient ω ($0 < \omega < 2$)
BiCGSafe	<code>-i {bicgsafe 13}</code>		
CR	<code>-i {cr 14}</code>		
BiCR	<code>-i {bicr 15}</code>		
CRS	<code>-i {crs 16}</code>		
BiCRSTAB	<code>-i {bicrstab 17}</code>		
GPBiCR	<code>-i {gpbicr 18}</code>		
BiCRSafe	<code>-i {bicrsafe 19}</code>		
FGMRES(m)	<code>-i {fgmres 20}</code>	<code>-restart [40]</code>	Restart value m
IDR(s)	<code>-i {idrs 21}</code>	<code>-irestart [2]</code>	Restart value s
MINRES	<code>-i {minres 22}</code>		

Specifying Preconditioners (Default: `-p none`)

Preconditioner	Option	Auxiliary Options	
None	<code>-p {none 0}</code>		
Jacobi	<code>-p {jacobi 1}</code>		
ILU(k)	<code>-p {ilu 2}</code>	<code>-ilu_fill [0]</code>	Fill level k
SSOR	<code>-p {ssor 3}</code>	<code>-ssor_w [1.0]</code>	Relaxation coefficient ω ($0 < \omega < 2$)
Hybrid	<code>-p {hybrid 4}</code>	<code>-hybrid_i [sor]</code>	Linear equations solver
		<code>-hybrid_maxiter [25]</code>	Maximum number of iterations
		<code>-hybrid_tol [1.0e-3]</code>	Convergence criterion
		<code>-hybrid_w [1.5]</code>	Relaxation coefficient ω for SOR ($0 < \omega < 2$)
		<code>-hybrid_ell [2]</code>	Degree l of BiCGSTAB(l)
		<code>-hybrid_restart [40]</code>	Restart values for GMRES and Orthomin
I+S	<code>-p {is 5}</code>	<code>-is_alpha [1.0]</code>	Parameter α for preconditioner of $I + \alpha S^{(m)}$ type
		<code>-is_m [3]</code>	Parameter m for preconditioner of $I + \alpha S^{(m)}$ type
SAINV	<code>-p {sainv 6}</code>	<code>-sainv_drop [0.05]</code>	Drop criterion
SA-AMG	<code>-p {saamg 7}</code>	<code>-saamg_unsym [false]</code>	Selects unsymmetric version (Matrix structure must be symmetric)
		<code>-saamg_theta [0.05 0.12]</code>	Drop criterion $a_{ij}^2 \leq \theta^2 a_{ii} a_{jj} $ (symmetric or unsymmetric)
Crout ILU	<code>-p {iluc 8}</code>	<code>-iluc_drop [0.05]</code>	Drop criterion
		<code>-iluc_rate [5.0]</code>	Ratio of maximum fill-in
ILUT	<code>-p {ilut 9}</code>	<code>-ilut_drop [0.05]</code>	Drop criterion
		<code>-ilut_rate [5.0]</code>	Ratio of maximum fill-in
Additive Schwarz	<code>-adds true</code>	<code>-adds_iter [1]</code>	Number of iterations

Other Options

Option	
<code>-maxiter [1000]</code>	Maximum number of iterations
<code>-tol [1.0e-12]</code>	Convergence criterion
<code>-print [0]</code>	Display of the residual
	<code>-print {none 0}</code> None
	<code>-print {mem 1}</code> Saves the residual history in memory
	<code>-print {out 2}</code> Displays the residual history
	<code>-print {all 3}</code> Saves the residual history and displays it on the screen
<code>-scale [0]</code>	Scaling (The result will overwrite the original matrix and vectors)
	<code>-scale {none 0}</code> No scaling
	<code>-scale {jacobi 1}</code> Jacobi scaling $D^{-1}Ax = D^{-1}b$ (D represents the diagonal of $A = (a_{ij})$)
	<code>-scale {symm_diag 2}</code> Diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$ ($D^{-1/2}$ represents a diagonal matrix with $1/\sqrt{a_{ii}}$ as diagonal)
<code>-initx_zeros [true]</code>	Behavior of the initial vector x_0
	<code>-initx_zeros {false 0}</code> Given values
	<code>-initx_zeros {true 1}</code> All elements are set to 0
<code>-omp_num_threads [t]</code>	Number of threads (t represents the maximum number of threads)
<code>-storage [0]</code>	Matrix storage format
<code>-storage_block [2]</code>	Block size of BSR and BSC

Precision (Default: `-f double`)

Precision	Option	Auxiliary Options
DOUBLE	<code>-f {double 0}</code>	
QUAD	<code>-f {quad 1}</code>	

6.4.4 lis_solver_set_optionC

```
C      int lis_solver_set_optionC(LIS_SOLVER solver)
Fortran subroutine lis_solver_set_optionC(LIS_SOLVER solver, integer ierr)
```

Description

Set the options for the solver on command line

Input

None

Output

<code>solver</code>	Solver
<code>ierr</code>	Return code

6.4.5 lis_solve

```
C      int lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver)
Fortran subroutine lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
                             LIS_SOLVER solver, integer ierr)
```

Description

Solve the linear equation $Ax = b$ with the specified solver

Input

A	Coefficient matrix
b	Right hand side vector
x	Initial vector
solver	Solver

Output

x	Solution
solver	Number of iterations, execution time, etc.
ierr	Return code (0)

6.4.6 lis_solve_kernel

```
C      int lis_solve_kernel(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,  
                          LIS_SOLVER solver, LIS_PRECON, precon)  
Fortran subroutine lis_solve_kernel(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,  
                                  LIS_SOLVER solver, LIS_PRECON precon, integer ierr)
```

Description

Solve the linear equation $Ax = b$ with the specified solver and the predefined preconditioner

Input

A	Coefficient matrix
b	Right hand side vector
x	Initial vector
solver	Solver
precon	Preconditioner

Output

x	Solution
solver	Number of iterations, execution time, etc.
ierr	Return code (0)

6.4.7 lis_solver_get_status

```
C      int lis_solver_get_status(LIS_SOLVER solver, int *status)
Fortran subroutine lis_solver_get_status(LIS_SOLVER solver, integer status,
                                         integer ierr)
```

Description

Get the status from the solver

Input

solver	Solver
--------	--------

Output

status	Number of iterations
ierr	Return code

6.4.8 lis_solver_get_iters

```
C      int lis_solver_get_iters(LIS_SOLVER solver, int *iters)
Fortran subroutine lis_solver_get_iters(LIS_SOLVER solver, integer iters,
                                         integer ierr)
```

Description

Get the number of iterations from the solver

Input

solver	Solver
--------	--------

Output

iters	Number of iterations
ierr	Return code

6.4.9 lis_solver_get_itersex

```
C      int lis_solver_get_itersex(LIS_SOLVER solver, int *iters, int *iters_double,  
                                int *iters_quad)  
Fortran subroutine lis_solver_get_itersex(LIS_SOLVER solver, integer iters,  
                                         integer iters_double, integer iters_quad, integer ierr)
```

Description

Get the number of iterations from the solver

Input

<code>solver</code>	Solver
---------------------	--------

Output

<code>iters</code>	Number of iterations
<code>iters_double</code>	Number of double precision iterations
<code>iters_quad</code>	Number of quadruple precision iterations
<code>ierr</code>	Return code

6.4.10 lis_solver_get_time

```
C      int lis_solver_get_time(LIS_SOLVER solver, double *times)  
Fortran subroutine lis_solver_get_time(LIS_SOLVER solver, real*8 times,  
                                       integer ierr)
```

Description

Get the execution time from the solver

Input

<code>solver</code>	Solver
---------------------	--------

Output

<code>times</code>	Time in seconds for execution
<code>ierr</code>	Return code

6.4.11 lis_solver_get_timeex

```
C      int lis_solver_get_timeex(LIS_SOLVER solver, double *times, double *itimes,
                                double *ptimes, double *p_c_times, double *p_i_times)
Fortran subroutine lis_solver_get_timeex(LIS_SOLVER solver, real*8 times,
                                         real*8 itimes, real*8 ptimes, real*8 p_c_times, real*8 p_i_times,
                                         integer ierr)
```

Description

Get the execution time from the solver

Input

<code>solver</code>	Solver
---------------------	--------

Output

<code>times</code>	Total time in seconds
<code>itimes</code>	Time in seconds for iteration
<code>ptimes</code>	Time in seconds for preconditioning
<code>p_c_times</code>	Time in seconds for creating preconditioner
<code>p_i_times</code>	Time in seconds for iteration in preconditioner
<code>ierr</code>	Return code

6.4.12 lis_solver_get_residualnorm

```
C      int lis_solver_get_residualnorm(LIS_SOLVER solver, LIS_REAL *residual)
Fortran subroutine lis_solver_get_residualnorm(LIS_SOLVER solver, LIS_REAL residual,
                                              integer ierr)
```

Description

Calculate the relative residual norm $\|b - Ax\|_2 / \|b\|_2$ from solution x

Input

<code>solver</code>	Solver
---------------------	--------

Output

<code>residual</code>	Relative residual norm $\ b - Ax\ _2 / \ b\ _2$
<code>ierr</code>	Return code

6.4.13 lis_solver_get_rhistory

```
C      int lis_solver_get_rhistory(VECTOR v)
Fortran subroutine lis_solver_get_rhistory(LIS_VECTOR v, integer ierr)
```

Description

Store the residual norm history of the solver

Input

None

Output

<code>v</code>	Vector
<code>ierr</code>	Return code

Note

The vector v must be created in advance with the function `lis_vector_create`. When the vector v is shorter than the residual history, it stores the residual history in order to the vector v .

6.4.14 lis_solver_get_solver

```
C      int lis_solver_get_solver(LIS_SOLVER solver, int *nsol)
Fortran subroutine lis_solver_get_solver(LIS_SOLVER solver, integer nsol,
      integer ierr)
```

Description

Get the solver number from the solver

Input

`solver` Solver

Output

`nsol` Solver number

`ierr` Return code

Note

The number of the solver is as follows:

Solver	Number	Solver	Number
CG	1	SOR	12
BiCG	2	BiCGSafe	13
CGS	3	CR	14
BiCGSTAB	4	BiCR	15
BiCGSTAB(l)	5	CRS	16
GPBiCG	6	BiCRSTAB	17
TFQMR	7	GPBiCR	18
Orthomin(m)	8	BiCRSafe	19
GMRES(m)	9	FGMRES(m)	20
Jacobi	10	IDR(s)	21
Gauss-Seidel	11	MINRES	22

6.4.15 lis_get_solvername

```
C      int lis_get_solvername(int nsol, char *name)
Fortran subroutine lis_get_solvername(integer nsol, character name, integer ierr)
```

Description

Get the solver name from the solver number

Input

`nsol` Solver number

Output

`name` Solver name

`ierr` Return code

6.5 Solving Eigenvalue Problems

6.5.1 lis_esolver_create

```
C      int lis_esolver_create(LIS_ESOLVER *esolver)
Fortran subroutine lis_esolver_create(LIS_ESOLVER esolver, integer ierr)
```

Description

Create the eigensolver

Input

None

Output

esolver	Eigensolver
ierr	Return code

Note

esolver has the information on the eigensolver, the preconditioner, etc.

6.5.2 lis_esolver_destroy

```
C      int lis_esolver_destroy(LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_destroy(LIS_ESOLVER esolver, integer ierr)
```

Description

Destroy the eigensolver

Input

esolver	Eigensolver to be destroyed
---------	-----------------------------

Output

ierr	Return code
------	-------------

6.5.3 lis_esolver_set_option

```
C      int lis_esolver_set_option(char *text, LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_set_option(character text, LIS_ESOLVER esolver,
      integer ierr)
```

Description

Set the options for the eigensolver

Input

text Command line options

Output

esolver Eigensolver

ierr Return code

Note

The table below shows the available command line options, where **-e {pi|1}** means **-e pi** or **-e 1** and **-emaxiter [1000]** indicates that **-emaxiter** defaults to 1,000.

Specifying Eigensolvers (Default: **-e pi**)

Method	Option	Auxiliary Options	
Power Iteration	-e {pi 1}		
Inverse Iteration	-e {ii 2}	-i [bicg]	Linear solver
Approximate Inverse Iteration	-e {aai 3}		
Rayleigh Quotient Iteration	-e {rqi 4}	-i [bicg]	Linear solver
Subspace Iteration	-e {si 5}	-ss [2]	Size of subspace
		-m [0]	Mode number
Lanczos Iteration	-e {li 6}	-ss [2]	Size of subspace
		-m [0]	Mode number
Conjugate Gradient	-e {cg 7}		
Conjugate Residual	-e {cr 8}		

Specifying Preconditioners (Default: -p ilu)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	Fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	Relaxation coefficient ω ($0 < \omega < 2$)
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	Linear equations solver
		-hybrid_maxiter [25]	Maximum number of iterations
		-hybrid_tol [1.0e-3]	Convergence criterion
		-hybrid_w [1.5]	Relaxation coefficient ω for SOR ($0 < \omega < 2$)
		-hybrid_ell [2]	Degree l of BiCGSTAB(l)
		-hybrid_restart [40]	Restart values for GMRES and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	Parameter α for preconditioner of $I + \alpha S^{(m)}$ type
		-is_m [3]	Parameter m for preconditioner of $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05]	Drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Selects unsymmetric version (Matrix structure must be symmetric)
		-saamg_theta [0.05 0.12]	Drop criterion $a_{ij}^2 \leq \theta^2 a_{ii} a_{jj} $ (symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	Drop criterion
		-iluc_rate [5.0]	Ratio of maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05]	Drop criterion
		-ilut_rate [5.0]	Ratio of maximum fill-in
Additive Schwarz	-adds true	-adds_iter [1]	Number of iterations

Other Options

Option	
<code>-emaxiter [1000]</code>	Maximum number of iterations
<code>-etol [1.0e-12]</code>	Convergence criterion
<code>-eprint [0]</code>	Display of the residual
	<code>-eprint {none 0}</code> None
	<code>-eprint {mem 1}</code> Saves the residual history in memory
	<code>-eprint {out 2}</code> Displays the residual history
	<code>-eprint {all 3}</code> Saves the residual history and displays it on the screen
<code>-ie [ii]</code>	Inner eigensolver used in Lanczos Iteration or Subspace Iteration
	<code>-ie {pi 1}</code> Power Iteration (Subspace Iteration only)
	<code>-ie {ii 2}</code> Inverse Iteration
	<code>-ie {aii 3}</code> Approximate Inverse Iteration
	<code>-ie {rqi 4}</code> Rayleigh Quotient Iteration
<code>-shift [0.0]</code>	Amount of shift
<code>-initx_ones [true]</code>	Behavior of the initial vector x_0
	<code>-initx_ones {false 0}</code> Given values
	<code>-initx_ones {true 1}</code> All elements are set to 1
<code>-omp_num_threads [t]</code>	Number of threads (t represents the maximum number of threads)
<code>-estorage [0]</code>	Matrix storage format
<code>-estorage_block [2]</code>	Block size of BSR and BSC

Precision (Default: `-ef double`)

Precision	Option	Auxiliary Options
DOUBLE	<code>-ef {double 0}</code>	
QUAD	<code>-ef {quad 1}</code>	

6.5.4 lis_esolver_set_optionC

```
C      int lis_esolver_set_optionC(LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_set_optionC(LIS_ESOLVER esolver, integer ierr)
```

Description

Set the options for the eigensolver on command line

Input

None

Output

esolver	Eigensolver
ierr	Return code

6.5.5 lis_solve

```
C      int lis_solve(LIS_MATRIX A, LIS_VECTOR x,
                   LIS_REAL evalue, LIS_ESOLVER esolver)
Fortran subroutine lis_solve(LIS_MATRIX A, LIS_VECTOR x,
                           LIS_REAL evalue, LIS_ESOLVER esolver, integer ierr)
```

Description

Solve the eigenvalue problem $Ax = \lambda x$ with the specified eigensolver

Input

A	Matrix
x	Initial vector
esolver	Eigensolver

Output

evalue	Eigenvalue of mode specified by -m [0] option
x	Associated eigenvector
esolver	Number of iterations, execution time, etc.
ierr	Return code (0)

6.5.6 lis_esolver_get_status

```
C      int lis_esolver_get_status(LIS_ESOLVER esolver, int *status)
Fortran subroutine lis_esolver_get_status(LIS_ESOLVER esolver, integer status,
      integer ierr)
```

Description

Get the status from the eigensolver

Input

esolver	Eigensolver
---------	-------------

Output

status	Number of iterations
ierr	Return code

6.5.7 lis_esolver_get_iters

```
C      int lis_esolver_get_iters(LIS_ESOLVER esolver, int *iters)
Fortran subroutine lis_esolver_get_iters(LIS_ESOLVER esolver, integer iters,
      integer ierr)
```

Description

Get the number of iterations from the eigensolver

Input

esolver	Eigensolver
---------	-------------

Output

iters	Number of iterations
ierr	Return code

6.5.8 lis_esolver_get_itersex

```
C      int lis_esolver_get_itersex(LIS_ESOLVER esolver, int *iters,  
                                int *iters_double, int *iters_quad)  
Fortran subroutine lis_esolver_get_itersex(LIS_ESOLVER esolver, integer iters,  
                                integer iters_double, integer iters_quad, integer ierr)
```

Description

Get the number of iterations from the eigensolver

Input

esolver	Eigensolver
---------	-------------

Output

iters	Number of iterations
iters_double	Number of double precision iterations
iters_quad	Number of quadruple precision iterations
ierr	Return code

6.5.9 lis_esolver_get_time

```
C      int lis_esolver_get_time(LIS_ESOLVER esolver, double *times)  
Fortran subroutine lis_esolver_get_time(LIS_ESOLVER esolver, real*8 times,  
                                integer ierr)
```

Description

Get the execution time from the eigensolver

Input

esolver	Eigensolver
---------	-------------

Output

times	Time in seconds for execution
ierr	Return code

6.5.10 lis_esolver_get_timeex

```
C      int lis_esolver_get_timeex(LIS_ESOLVER esolver, double *times,
                                double *itimes, double *ptimes, double *p_c_times, double *p_i_times)
Fortran subroutine lis_esolver_get_timeex(LIS_ESOLVER esolver, real*8 times,
                                real*8 itimes, real*8 ptimes, real*8 p_c_times, real*8 p_i_times,
                                integer ierr)
```

Description

Get the execution time from the eigensolver

Input

esolver	Eigensolver
---------	-------------

Output

times	Total time in seconds
itimes	Time in seconds for iteration
ptimes	Time in seconds for preconditioning
p_c_times	Time in seconds for creating preconditioner
p_i_times	Time in seconds for iteration in preconditioner
ierr	Return code

6.5.11 lis_esolver_get_residualnorm

```
C      int lis_esolver_get_residualnorm(LIS_ESOLVER esolver, LIS_REAL *residual)
Fortran subroutine lis_esolver_get_residualnorm(LIS_ESOLVER esolver,
                                LIS_REAL residual, integer ierr)
```

Description

Calculate the relative residual norm $\|\lambda x - Ax\|_2/\lambda$ from eigenvector x

Input

esolver	Eigensolver
---------	-------------

Output

residual	Relative residual norm $\ \lambda x - Ax\ _2/\lambda$
ierr	Return code

6.5.12 lis_esolver_get_rhistory

```
C      int lis_esolver_get_rhistory(VECTOR v)
Fortran subroutine lis_esolver_get_rhistory(LIS_VECTOR v, integer ierr)
```

Description

Store the residual norm history of the eigensolver

Input

None

Output

v Vector

ierr Return code

Note

The vector v must be created in advance with the function `lis_vector_create`. When the vector v is shorter than the residual history, it stores the residual history in order to the vector v .

6.5.13 lis_esolver_get_evalues

```
C      int lis_esolver_get_evalues(LIS_ESOLVER esolver, LIS_VECTOR v)
Fortran subroutine lis_esolver_get_evalues(LIS_ESOLVER esolver,
      LIS_VECTOR v, integer ierr)
```

Description

Store the eigenvalues in the vector v

Input

esolver	Eigensolver
---------	-------------

Output

v	Vector which stores eigenvalues
ierr	Return code

Note

The vector v must be created in advance with the function `lis_vector_create`.

6.5.14 lis_esolver_get_evectors

```
C      int lis_esolver_get_evectors(LIS_ESOLVER esolver, LIS_MATRIX A)
Fortran subroutine lis_esolver_get_evectors(LIS_ESOLVER esolver,
      LIS_MATRIX A, integer ierr)
```

Description

Store the eigenvectors in the matrix A

Input

esolver	Eigensolver
---------	-------------

Output

A	Matrix in CRS format which stores eigenvectors
ierr	Return code

Note

The matrix A must be created in advance with the function `lis_matrix_create`.

6.5.15 lis_esolver_get_esolver

```
C      int lis_esolver_get_esolver(LIS_ESOLVER esolver, int *nesol)
Fortran subroutine lis_esolver_get_esolver(LIS_ESOLVER esolver, integer nesol,
      integer ierr)
```

Description

Get the eigensolver number from the eigensolver

Input

esolver Eigensolver

Output

nesol Eigensolver number

ierr Return code

Note

The number of the eigensolver is as follows:

Method	Number
Power Iteration	1
Inverse Iteration	2
Approximate Inverse Iteration	3
Rayleigh Quotient Iteration	4
Subspace Iteration	5
Lanczos Iteration	6
Conjugate Gradient	7
Conjugate Residual	8

6.5.16 lis_get_esolvername

```
C      int lis_get_esolvername(int nesol, char *ename)
Fortran subroutine lis_get_esolvername(integer nesol, character ename, integer ierr)
```

Description

Get the eigensolver name from the eigensolver number

Input

nesol Eigensolver number

Output

name Eigensolver name

ierr Return code

6.6 Operating External Files

6.6.1 lis_input

```
C      int lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, char *filename)
Fortran subroutine lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
      character filename, integer ierr)
```

Description

Read the matrix and vector data from the external file

Input

filename	Source file
----------	-------------

Output

A	Matrix in specified storage format
b	Right hand side vector
x	Solution
ierr	Return code

Note

The supported file formats are shown below:

- Matrix Market format (extended to allow vector data to be read)
- Harwell-Boeing format

6.6.2 lis_input_vector

```
C      int lis_input_vector(LIS_VECTOR v, char *filename)
Fortran subroutine lis_input_vector(LIS_VECTOR v, character filename, integer ierr)
```

Description

Read the vector data from the external file

Input

filename	Source file
----------	-------------

Output

v	Vector
ierr	Return code

Note

The following formats are supported:

- PLAIN format
- Matrix Market format

6.6.3 lis_input_matrix

```
C      int lis_input_matrix(LIS_MATRIX A, char *filename)
Fortran subroutine lis_input_matrix(LIS_MATRIX A, LIS_VECTOR x,
      character filename, integer ierr)
```

Description

Read the matrix data from the external file

Input

filename	Source file
----------	-------------

Output

A	Matrix in specified storage format
x	Solution
ierr	Return code

Note

The supported file formats are shown below:

- Matrix Market format (extended to allow vector data to be read)
- Harwell-Boeing format

6.6.4 lis_output

```
C      int lis_output(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, int format,
      char *filename)
Fortran subroutine lis_output(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
      integer format, character path, integer ierr)
```

Description

Write the matrix and vector data into the external file

Input

A	Matrix
b	Right hand side vector (If no vector is written to an external file, then NULL must be input.)
x	Solution (If no vector is written to an external file, then NULL must be input.)
format	File format
	LIS_FMT_MM Matrix Market format
filename	Destination file

Output

ierr	Return code
------	-------------

6.6.5 lis_output_vector

```
C      int lis_output_vector(LIS_VECTOR v, int format, char *filename)
Fortran subroutine lis_output_vector(LIS_VECTOR v, integer format,
      character filename, integer ierr)
```

Description

Write the vector data into the external file

Input

v	Vector	
format	File format	
	LIS_FMT_PLAIN	PLAIN format
	LIS_FMT_MM	Matrix Market format
filename	Destination file	

Output

ierr	Return code
------	-------------

6.6.6 lis_output_matrix

```
C      int lis_output_matrix(LIS_MATRIX A, int format, char *filename)
Fortran subroutine lis_output_matrix(LIS_MATRIX A, integer format, character path,
      integer ierr)
```

Description

Write the matrix data into the external file

Input

A	Matrix	
format	File format	
	LIS_FMT_MM	Matrix Market format
filename	Destination file	

Output

ierr	Return code
------	-------------

6.7 Other Functions

6.7.1 lis_initialize

```
C      int lis_initialize(int* argc, char** argv[])
Fortran subroutine lis_initialize(integer ierr)
```

Description

Initialize the execution environment

Input

<code>argc</code>	Number of command line arguments
<code>argv</code>	Command line argument

Output

<code>ierr</code>	Return code
-------------------	-------------

6.7.2 lis_finalize

```
C      void lis_finalize()
Fortran subroutine lis_finalize(integer ierr)
```

Description

Finalize the execution environment

Input

None

Output

<code>ierr</code>	Return code
-------------------	-------------

6.7.3 lis_wtime

```
C      double lis_wtime()
Fortran function lis_wtime()
```

Description

Measure the elapsed time

Input

None

Output

Elapsed time in seconds from given point is returned as double number

Note

To measure the processing time, call `lis_wtime` to get the starting time, call it again to get the ending time, and calculate the difference.

References

- [1] S. Fujino, M. Fujiwara and M. Yoshida. BiCGSafe method based on minimization of associate residual (in Japanese). Transactions of JSCES, Paper No.20050028, 2005. <http://save.k.u-tokyo.ac.jp/jscs/trans/trans2005/No20050028.pdf>.
- [2] T. Sogabe, M. Sugihara and S. Zhang. An Extension of the Conjugate Residual Method for Solving Nonsymmetric Linear Systems(in Japanese). Transactions of the Japan Society for Industrial and Applied Mathematics, Vol. 15, No. 3, pp. 445–460, 2005.
- [3] K. Abe, T. Sogabe, S. Fujino and S. Zhang. A Product-type Krylov Subspace Method Based on Conjugate Residual Method for Nonsymmetric Coefficient Matrices (in Japanese). IPSJ Transactions on Advanced Computing Systems, Vol. 48, No. SIG8(ACS18), pp. 11–21, 2007.
- [4] S. Fujino and Y. Onoue. Estimation of BiCRSafe method based on residual of BiCR method (in Japanese). IPSJ SIG Technical Report, 2007-HPC-111, pp. 25–30, 2007.
- [5] Y. Saad. A Flexible Inner-outer Preconditioned GMRES Algorithm. SIAM J. Sci. Stat. Comput., Vol. 14, pp. 461–469, 1993.
- [6] Y. Saad. ILUT: a dual threshold incomplete LU factorization. Numerical linear algebra with applications, Vol. 1, No. 4, pp. 387–402, 1994.
- [7] ITSOL: ITERATIVE SOLVERS package
<http://www-users.cs.umn.edu/~saad/software/ITSOL/index.html>.
- [8] N. Li, Y. Saad and E. Chow. Crout version of ILU for general sparse matrices. SIAM J. Sci. Comput., Vol. 25, pp. 716–728, 2003.
- [9] T. Kohno, H. Kotakemori and H. Niki. Improving the Modified Gauss-Seidel Method for Z-matrices. Linear Algebra and its Applications, Vol. 267, pp. 113–123, 1997.
- [10] A. Fujii, A. Nishida, and Y. Oyanagi. Evaluation of Parallel Aggregate Creation Orders : Smoothed Aggregation Algebraic Multigrid Method. High Performance Computational Science And Engineering, pp. 99–122, Springer, 2005.
- [11] K. Abe, S. Zhang, H. Hasegawa and R. Himeno. A SOR-base Variable Preconditioned CGR Method (in Japanese). Trans. JSIAM, Vol. 11, No. 4, pp. 157–170, 2001.
- [12] R. Bridson and W.-P. Tang. Refining an approximate inverse. J. Comput. Appl. Math., Vol. 123, pp. 293–306, 2000.
- [13] P. Sonnerfeld and M. B. van Gijzen. IDR(s): a family of simple and fast algorithms for solving large nonsymmetric systems of linear equations. SIAM J. Sci. Comput., Vol. 31, Issue 2, pp. 1035–1062, 2008.
- [14] A. Greenbaum. Iterative Methods for Solving Linear Systems. SIAM, 1997.
- [15] D. H. Bailey. A fortran-90 double-double library. <http://www.nersc.gov/~dhbailey/mpdist/mpdist.html>.
- [16] Y. Hida, X. S. Li and D. H. Bailey. Algorithms for quad-double precision floating point arithmetic. Proceedings of the 15th Symposium on Computer Arithmetic, pp.155–162, 2001.
- [17] T. Dekker. A floating-point technique for extending the available precision. Numerische Mathematik, vol.18 pp. 224–242, 1971.
- [18] A. V. Knyazev. Toward the Optimal Preconditioned Eigensolver: Locally Optimal Block Preconditioned Conjugate Gradient Method. SIAM J. Sci. Comput., Vol. 23, No. 2, pp. 517–541, 2001.

- [19] A. Nishida. Experience in Developing an Open Source Scalable Software Infrastructure in Japan. Lecture Notes in Computer Science 6017, Springer, pp. 87-98, 2010.
- [20] E. Suetomi and H. Sekimoto. Conjugate gradient like methods and their application to eigenvalue problems for neutron diffusion equation. Annals of Nuclear Energy, Vol. 18, No. 4, pp. 205–227, 1991.
- [21] D. E. Knuth. The Art of Computer Programming: Seminumerical Algorithms, vol.2. Addison-Wesley, 1969.
- [22] D. H. Bailey. High-Precision Floating-Point Arithmetic in Scientific Computation. Computing in Science and Engineering, Volume 7, Issue 3, pp. 54–61, IEEE, 2005.
- [23] Intel Fortran Compiler User’s Guide Vol I.
- [24] H. Kotakemori, A. Fujii, H. Hasegawa and A. Nishida. Implementation of Fast Quad Precision Operation and Acceleration with SSE2 for Iterative Solver Library (in Japanese). IPSJ Transactions on Advanced Computing Systems, Vol. 1, No. 1, pp. 73–84, 2008.
- [25] R. Barrett, et al. Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods. SIAM, 1994.
- [26] Z. Bai, et al. Templates for the Solution of Algebraic Eigenvalue Problems. SIAM, 2000.
- [27] Y. Saad. SPARSKIT: a basic tool kit for sparse matrix computations, version 2, June 1994. <http://www.cs.umn.edu/~saad/software/SPARSKIT/sparskit.html>.
- [28] S. Balay, et al. PETSc users manual. Technical Report ANL-95/11, Argonne National Laboratory, August 2004.
- [29] R. S. Tuminaro, et al. Official Aztec user’s guide, version 2.1. Technical Report SAND99-8801J, Sandia National Laboratories, November 1999.
- [30] R. B. Lehoucq, D. C. Sorensen, and C. Yang. ARPACK Users’ Guide: Solution of Large-scale Eigenvalue Problems with implicitly-restarted Arnoldi Methods. SIAM, 1998.
- [31] R. Bramley and X. Wang. SPLIB: A library of iterative methods for sparse linear system. Technical report, Indiana University–Bloomington, 1995.
- [32] Matrix Market. <http://math.nist.gov/MatrixMarket>.

A File Formats

This section describes the file formats available for the library.

A.1 Extended Matrix Market Format

The Matrix Market format[32] is not designed to store vector data. For this library, it has been extended to handle vector data. Assume that the number of the nonzero elements for the matrix $A = (a_{ij})$ of $M \times N$ is L and that $a_{ij} = A(I, J)$. The format is as follows:

```
%%MatrixMarket matrix coordinate real general <-- Header
%
% | Comment lines with 0 or more lines
%
% <--+
M N L B X <-- Numbers of rows, columns, and
I1 J1 A(I1,J1) <--+ nonzero elements (0 or 1) (0 or 1)
I2 J2 A(I2,J2) | Row and column number values
. . . | The index is one origin
IL JL A(IL,JL) <--+
I1 B(I1) <--+
I2 B(I2) | Exists only when B=1
. . . | Row number value
IM B(IM) <--+
I1 X(I1) <--+
I2 X(I2) | Exists only when X=1
. . . | Row number value
IM X(IM) <--+
```

The extended Matrix Market format for the matrix A and the vector b in Equation (A.1) is as follows:

$$A = \begin{pmatrix} 2 & 1 & & \\ 1 & 2 & 1 & \\ & 1 & 2 & 1 \\ & & 1 & 2 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \quad (\text{A.1})$$

```
%%MatrixMarket matrix coordinate real general
4 4 10 1 0
1 2 1.00e+00
1 1 2.00e+00
2 3 1.00e+00
2 1 1.00e+00
2 2 2.00e+00
3 4 1.00e+00
3 2 1.00e+00
3 3 2.00e+00
4 4 2.00e+00
4 3 1.00e+00
1 0.00e+00
2 1.00e+00
3 2.00e+00
4 3.00e+00
```

A.2 Harwell-Boeing Format

The Harwell-Boeing format inputs and outputs the matrix in the CCS storage format. Assume that the array `value` stores the values of the nonzero elements of the matrix A , the array `index` stores the row indices of the nonzero elements and the array `ptr` stores pointers to the top of each column in the arrays `value` and `index`. The format is as follows:

```

Line 1 (A72,A8)
  1 - 72 Title
  73 - 80 Key
Line 2 (5I14)
  1 - 14 Total number of lines excluding header
  15 - 28 Number of lines for ptr
  29 - 42 Number of lines for index
  43 - 56 Number of lines for value
  57 - 70 Number of lines for right hand side vectors
Line 3 (A3,11X,4I14)
  1 - 3 Matrix type
      Col.1: R Real matrix
            C Complex matrix (Not supported)
            P Pattern only (Not supported)
      Col.2: S Symmetric
            U Unsymmetric
            H Hermitian (Not supported)
            Z Skew symmetric (Not supported)
            R Rectangular (Not supported)
      Col.3: A Assembled
            E Elemental matrices (Not supported)
  4 - 14 Blank space
  15 - 28 Number of rows
  29 - 42 Number of columns
  43 - 56 Number of nonzero elements
  57 - 70 0
Line 4 (2A16,2A20)
  1 - 16 Format for ptr
  17 - 32 Format for index
  33 - 52 Format for value
  53 - 72 Format for right hand side vectors
Line 5 (A3,11X,2I14) Only presents if there are right hand side vectors
  1   Right hand side vector type
      F for full storage
      M for same format as matrix (Not supported)
  2   G if a starting vector is supplied
  3   X if an exact solution is supplied
  4 - 14 Blank space
  15 - 28 Number of right hand side vectors
  29 - 42 Number of nonzero elements

```

The Harwell-Boeing format for the matrix A and the vector b in Equation (A.1) is as follows:

```

1-----10-----20-----30-----40-----50-----60-----70-----80
Harwell-Boeing format sample                                     Lis
      8              1              1              4              2
RUA              4              4              10             4
(11i7)          (13i6)          (3e26.18)          (3e26.18)
F              1              0
      1      3      6      9
      1      2      1      2      3      2      3      4      3      4
2.0000000000000000E+00  1.0000000000000000E+00  1.0000000000000000E+00
2.0000000000000000E+00  1.0000000000000000E+00  1.0000000000000000E+00
2.0000000000000000E+00  1.0000000000000000E+00  1.0000000000000000E+00
2.0000000000000000E+00
0.0000000000000000E+00  1.0000000000000000E+00  2.0000000000000000E+00
3.0000000000000000E+00

```

A.3 Extended Matrix Market Format for Vectors

The Matrix Market format[32] has been extended to store vector data. Assume that the vector $b = (b_i)$ is a vector of order N and that $b_i = B(I)$. The format is as follows:

```
%%MatrixMarket vector coordinate real general <-- Header
% <--+
% | Comment lines with 0 or more lines
% <--+
N <-- Number of rows
I1 B(I1) <--+
I2 B(I2) | Row number value
. . . | The index is one origin
IN B(IN) <--+
```

The extended Matrix Market format for the vector b in Equation (A.1) is as follows:

```
%%MatrixMarket vector coordinate real general
4
1 0.00e+00
2 1.00e+00
3 2.00e+00
4 3.00e+00
```

A.4 PLAIN Format for Vectors

The PLAIN format for vectors is designed to write vector values in order. Assume that the vector $b = (b_i)$ is a vector of order N and that $b_i = B(I)$. The format is as follows:

```
B(1) <--+
B(2) | Vector value
. . . |
B(N) <--+
```

The PLAIN format for the vector b in Equation (A.1) is as follows:

```
0.00e+00
1.00e+00
2.00e+00
3.00e+00
```