Parallel Matrix Distribution Library for Sparse Matrix Solvers

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Target Problem

- Parallel matrix distribution library for sparse matrix solvers which can deal with
  - Load-balancing
  - Ordering of the distributed matrix
- Usability of the library
  - Easiness to use and to create various load-balancing methods

Outline

- Background and Objective
- Matrix Distribution Library
  - Library Interface
- Load-balancing Examples using the library
- Numerical Tests of the examples
- Evaluation and Conclusion
- Future Work

Background and Objective

- Few libraries enable users to distribute the sparse matrix
  - Because user's data structure cannot be specified to one matrix format.
- Few libraries manage the ordering of the rows of the distributed matrix.

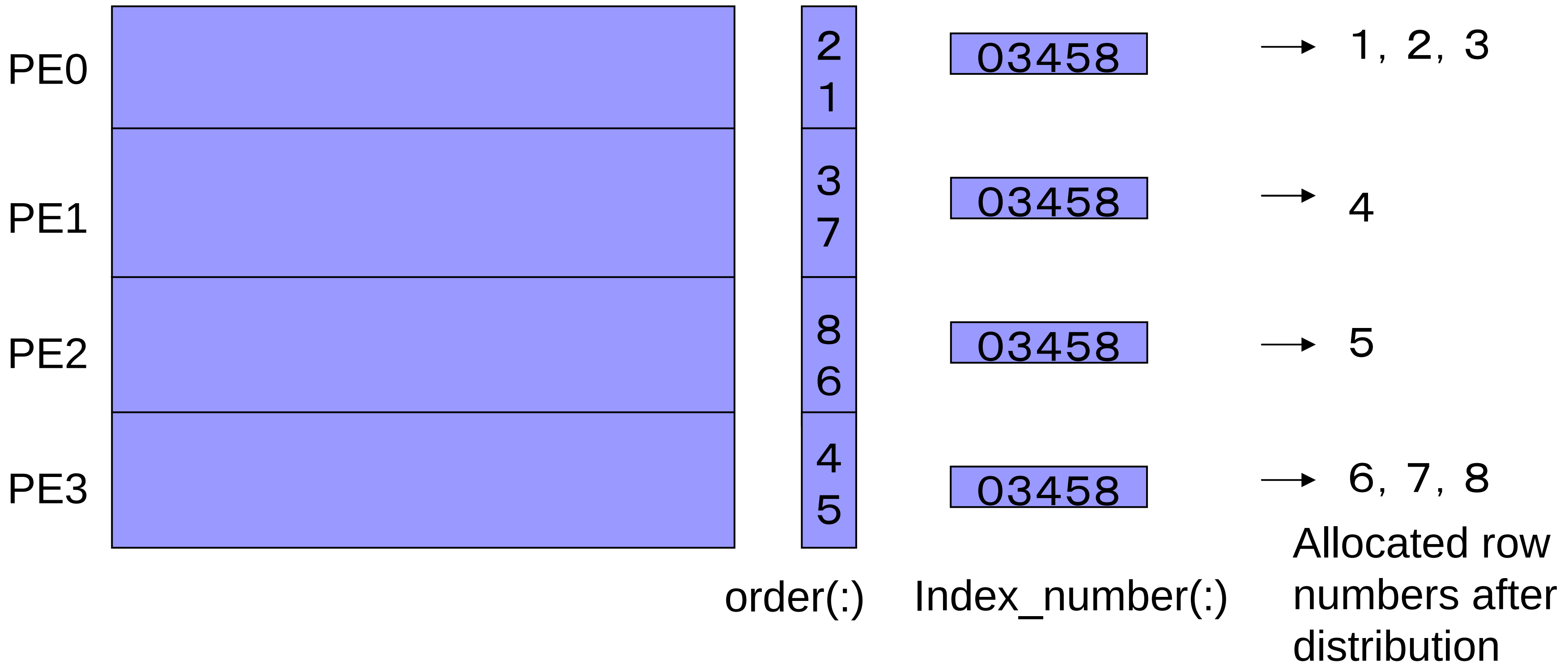
- Especially for sparse matrix solvers, the CRS format is often used

Objective: Consider and implement such library based on the distributed CRS format.

Features of the Matrix Distribution Library

- Any distribution and any order of the rows of the matrix can be specified for re-distribution.
- The library doesn’t use the particular matrix format other than distributed CRS format.
  - So users can use the distributed matrix easily.
  - If necessary, re-distribution can be done repeatedly.
- Memory is managed by users.
  - Users know how much memory is needed to reorder and redistribute the matrix before the matrix distribution.

Library interface which specifies both distribution and ordering



order(:) shows new row number for each row after reordering

index_number(:) has the last row numbers on PEs after distribution

Implementation

- Copy-and-Delete strategy is taken.
- Execution steps as follows.
  - The user call the library subroutine to calculate the size of the new reordered matrix, and then allocate the memory.
  - The library redistribute the sparse matrix there.

Library Subroutines

- set_order(some arguments..)
  - stores the Ordering information in the arrays of the arguments.
- size_new_mat(some arguments..)
  - calculates the size of new distributed matrix size.
- assign_new_mat(some arguments..)
  - has the arguments of user allocated space, and assigns new distributed matrix there.
- distribute_vector(some arguments..)
  - distributes vector in the way specified in the arguments.

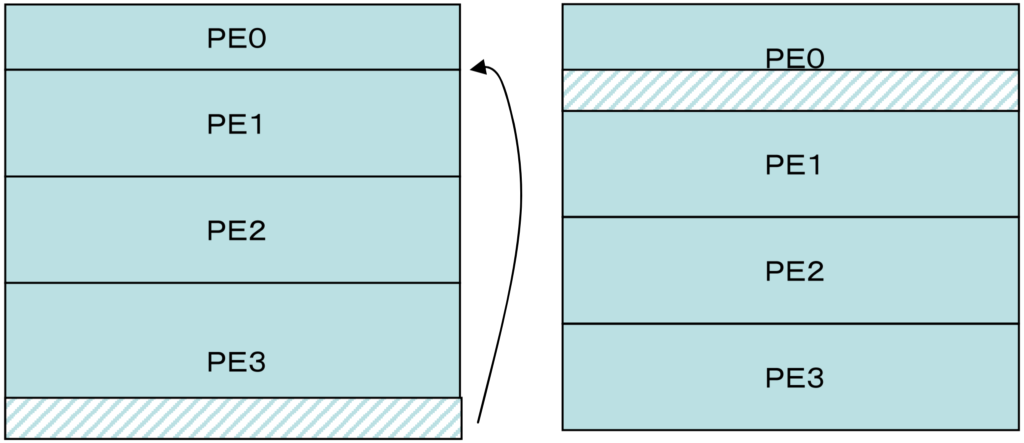
Various Matrix Distribution using this library as examples

The case: According to the weight on each PE, the sparse matrix is distributed

- Distribution with lowest communication cost
  - lowest comm
- Distribution based on ParMETIS
  - metis dist
- Distribution with no reordering
  - constant order

Distribution method1: lowest comm

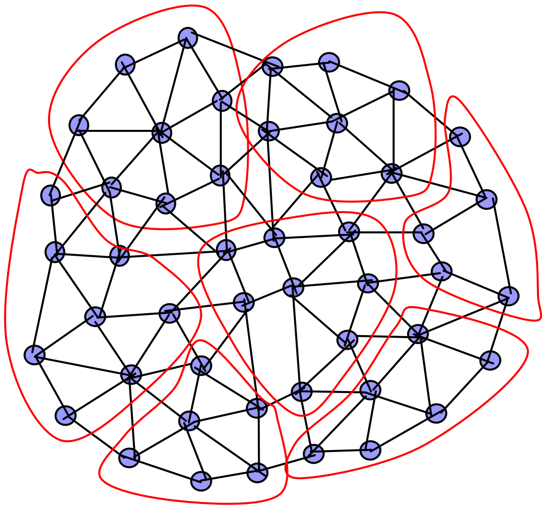
Mat.A



- Communication occurs between the PEs with too few or too many rows of the matrix.
  - Communication cost in distribution is low
  - Order of the rows changes after distribution

Distribution method2：metis dist

Mat A ⇒

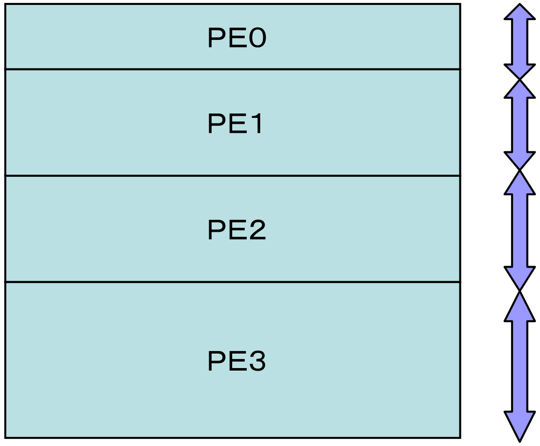


1. ParMETIS partitions the graph
2. Numbering in each partitioned domain
3. Matrix distribution

- Matrix distribution based on ParMETIS
  - The number of edges between domains is expected to be minimized
  - The calculation cost of both ParMETIS and generation of the graph for ParMETIS is needed.

Distribution method3：constant order

Mat A



- The order of the matrix rows is fixed, but the width of the rows on each PE changes.

Numerical Test

- Objectives:
  - Are the ordering and the distribution of problem matrix the key elements for performance?
  - What happened on performance, if the sparse matrix is distributed repeatedly?
  - How different are the performance behaviors on various matrix distribution methods?

Numerical Test

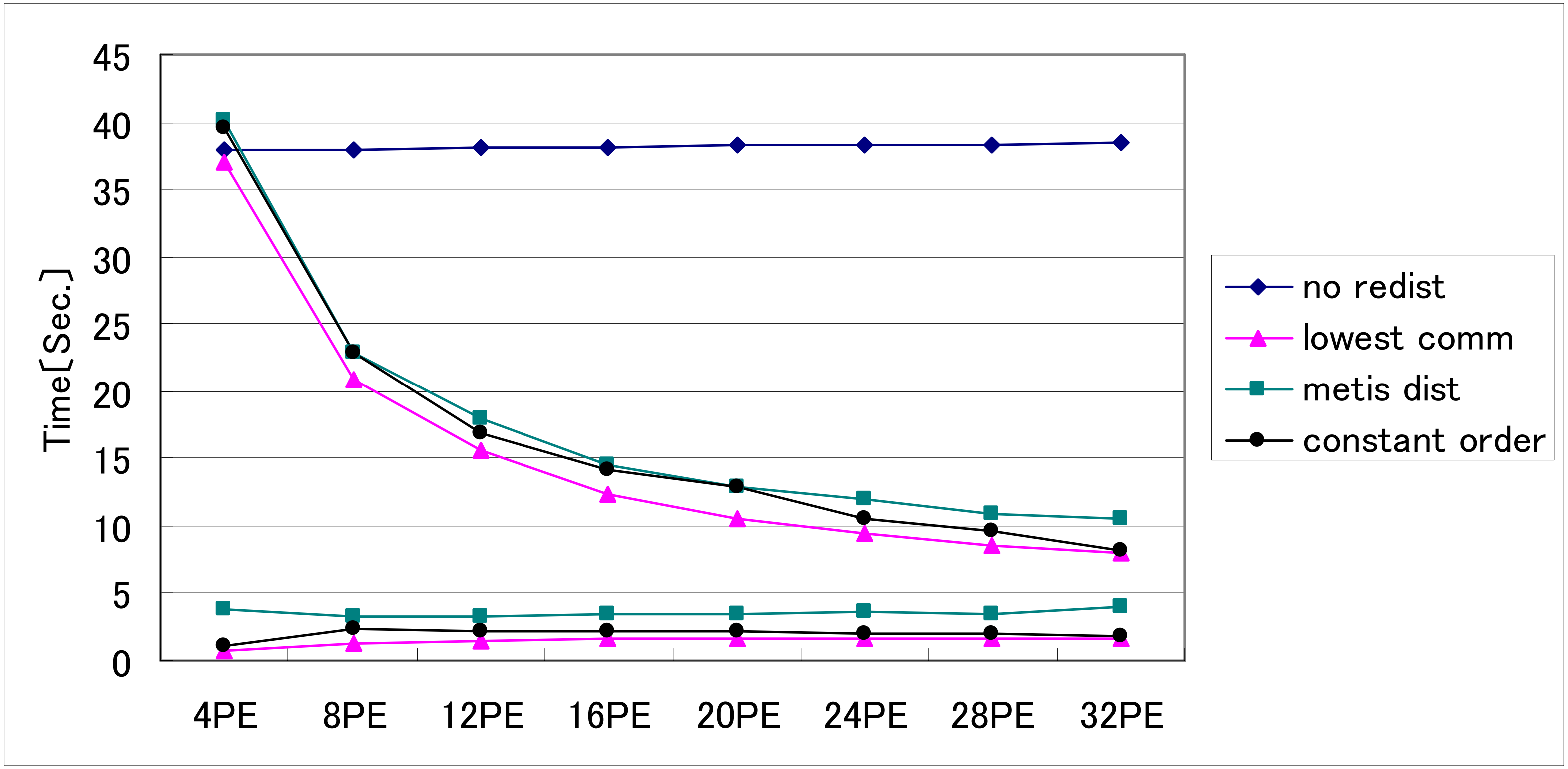
- Poisson problem on Cubic domain
  - 100x100x100:
- Environment：
  - 16 nodes (Xeon　2.8GHz x2 memory1GB）connected by 1000Base
  - LAM/MPI and Intel Fortran Compiler are used

$$\partial / \partial x\left(\partial p / \partial x\right) + \partial / \partial y\left(\partial p / \partial y\right) + \partial / \partial z\left(\partial p / \partial z\right) = 0$$

$$\begin{cases} \text{Xmin} : \partial p / \partial x = 10.0 \\ \text{Ymin} : \partial p / \partial y = 5.0 \\ \text{Zmax} : \partial p / \partial z = 1.0 \\ \text{Zmin} : p = 0.0 \end{cases}$$

Numerical Test 1

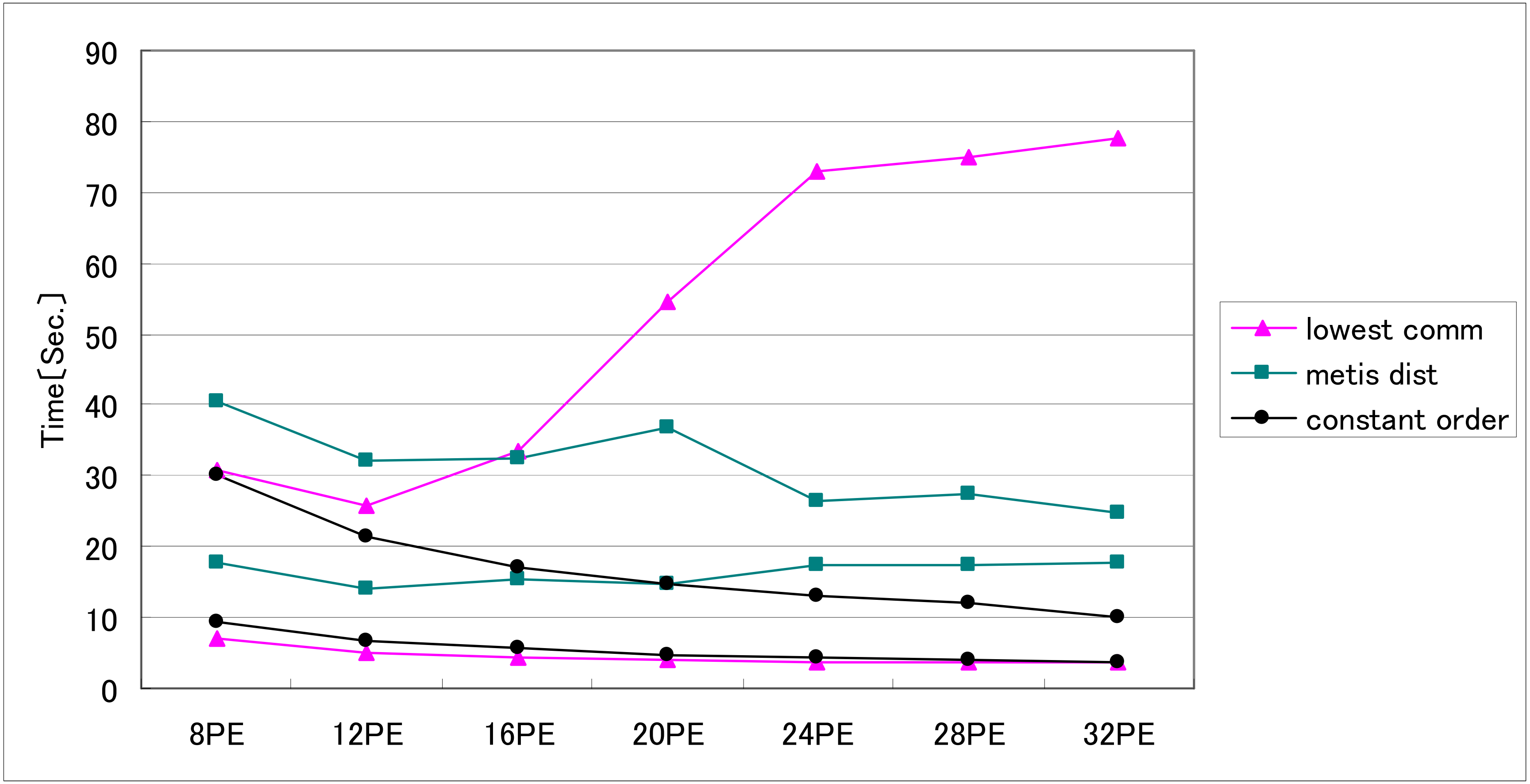
Initially, the problem matrix is distributed at first 4PEs. Each PE has 250000 rows. After matrix re-distribution, ICCG solver is called.



no redist: matrix distribution isn't done.

Numerical Test 2

Initially, the problem matrix is distributed at first 4PEs. After 5-time matrix distribution with different weights, ICCG solver is called.



Evaluation for distribution methods

- lowest comm
  - The cost of the distribution is low
  - Repeated matrix distribution results in the degradation of the ordering
- metis dist
  - The cost of the distribution is high
  - Communication tables are expected to be minimized
- constant order
  - The cost of the distribution is low
  - The degradation of the ordering is small even after the repeated distribution

Summary and Conclusion

- In the case that repeated matrix distribution is needed, the matrix distribution method should be considered carefully.
  - The degradation of the ordering often occurs.
  - The cost of distribution may influence the overall performance.
- Various matrix distribution routines can be made using this library
  - In order to create new matrix distribution method, the user have to create only one subroutine which specifies the ordering and the distribution.

Future Work

- Various reordering routines and matrix distribution routines will be created
- Other sparse matrix data structure than distributed CRS format will be added to the library