

## Lis 1.2.12 User's Manual

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## **0 Additions and changes from Lis 1.1**

### **0.1 Additions**

1. Added support of eigensolvers for real matrices

### **0.2 Changes**

1. Changed some specifications
  - (a) Changed the names of `lis_output_residual_history()` and `lis_get_residual_history()` to `lis_solver_output_rhistory()` and `lis_solver_get_rhistory()`, respectively.

# 1 Introduction

Lis, a Library of Iterative Solvers for linear systems, is a numerical library written in C and Fortran for solving the linear equation

$$Ax = b$$

or the eigenvalue problem

$$Ax = \lambda x$$

using iterative methods. Lis has the following features:

- 22 linear equation solvers, 7 eigensolvers, and 10 preconditioners are supported
- 11 sparse matrix storage formats are supported
- Both the sequential and parallel codes are supported by the common interface
- Both the double and quadruple precision operations are supported by the common interface

The solvers supported here are listed in Table 1 and 2, and the preconditioners are listed in Table 3. The matrix storage formats are listed in Table 4.

Table 1: Linear Equation Solvers

CG	CR
BiCG	BiCR[2]
CGS	CRS[3]
BiCGSTAB	BiCRSTAB[3]
GPBiCG	GPBiCR[3]
BiCGSafe[1]	BiCRSafe[4]
BiCGSTAB(1)	TFQMR
Jacobi	Orthomin(m)
Gauss-Seidel	GMRES(m)
SOR	FGMRES(m)[5]
IDR(s)[13]	MINRES[14]

Table 2: Eigensolvers

Power Iteration
Inverse Iteration
Approximate Inverse Iteration
Conjugate Gradient[18, 19]
Lanczos Iteration
Subspace Iteration
Conjugate Residual [20]

Table 3: Preconditioners

Jacobi
SSOR
ILU(k)
ILUT[6, 7]
Crout ILU[8, 7]
I+S[9]
SA-AMG[10]
hybrid[11]
SAINV[12]
additive Schwarz
User defined

Table 4: Matrix Storage Formats

Compressed Row Storage	(CRS)
Compressed Column Storage	(CCS)
Modified Compressed Sparse Row	(MSR)
Diagonal	(DIA)
Ellpack-Itpack generalized diagonal	(ELL)
Jagged Diagonal	(JDS)
Block Sparse Row	(BSR)
Block Sparse Column	(BSC)
Variable Block Row	(VBR)
Dense	(DNS)
Coordinate	(COO)

## 2 Installation

This section describes the instructions for installing and testing Lis. We assume Lis being installed on a Linux cluster.

## 2.1 Requirements

Installation of Lis requires a C compiler. Fortran interface requires a compiler which supports FORTRAN 77. AMG preconditioner requires a compiler which supports Fortran 90. For parallel computing environments, OpenMP or MPI-1 is used. Lis has been tested on the environments shown in Table 5 (see also Table 7).

Table 5: Major Tested Platforms

C Compiler (Required)	OS
Intel C/C++ Compiler 7.0, 8.0, 9.0, 9.1, 10.1, 11.0	Linux
IBM XL C/C++ V7.0, 9.0	Linux AIX 5.3
Sun WorkShop 6 update 2, Sun ONE Studio 7 Sun Studio 11, Sun Studio 12	Solaris 9 Solaris 10
PGI C++ 6.0, 7.1	Linux
gcc 3.3, 4.1	Linux
Fortran Compiler (Option)	OS
Intel Fortran Compiler 8.1, 9.0, 9.1, 10.1, 11.0	Linux
IBM XL Fortran V9.1, 11.1	Linux AIX 5.3
Sun WorkShop 6 update 2, Sun ONE Studio 7 Sun Studio 11, Sun Studio 12	Solaris 9 Solaris 10
PGI Fortran 6.0, 7.1	Linux
g77 3.3 gfortran 4.3 g95 0.91	Linux

## 2.2 Decompressing Archive

Enter the following command to decompress the archive, where (\$VERSION) represents the version:

```
>gunzip -c lis-($VERSION).tar.gz | tar xvf -
```

This command creates a directory `lis-($VERSION)` along with its subfolders as shown in Figure 1.

```
lis-($VERSION)
+ config
|  Configure files
+ include
|  Include files
+ src
|  Source files
+ test
  Test programs
```

Figure 1: Files contained in `lis-($VERSION).tar.gz`



## 2.3 Running Configure Script

Enter the following command to run the script:

- default: `>./configure`
- specifying the installation destination: `>./configure --prefix=<install-dir>`

Table 6 shows the option which can be specified for configuration. Table 7 shows the computer environment which can be specified by TARGET.

Table 6: Configuration Options

<code>--enable-omp</code>	Use OpenMP
<code>--enable-mpi</code>	Use MPI
<code>--enable-fortran</code>	Use Fortran API
<code>--enable-saamg</code>	Use SA-AMG preconditioner
<code>--enable-quad</code>	Use quadruple precision operation
<code>--prefix=&lt;install-dir&gt;</code>	Name of the installation target directory
<code>TARGET=&lt;target&gt;</code>	Computing environments
<code>CC=&lt;c_compiler&gt;</code>	C compiler
<code>CFLAGS=&lt;c_flags&gt;</code>	Compilation options for C compilers
<code>FC=&lt;fortran90_compiler&gt;</code>	Fortran 90 compiler
<code>FCFLAGS=&lt;fc_flags&gt;</code>	Compilation options for the Fortran 90 compiler
<code>LDFLAGS=&lt;ld_flags&gt;</code>	Link options

## 2.4 Running Make

In the `lis-($VERSION)` directory, run make by entering

```
>make
```

compilation is then performed.

## 2.5 Testing

Here, ensure that `make` (library compilation) has been successfully run.

To do so, in the `lis-($VERSION)` directory, enter as follows:

```
>make check
```

This command performs a test using the executable files created in the `lis-($VERSION)/test` directory. The test reads matrix and vector data from the MatrixMarket file `lis-($VERSION)/test/testmat.mtx` and writes the solution of the linear equation  $Ax = b$  obtained with the BiCG method into `lis-($VERSION)/test/sol.txt`, and the residual history into `lis-($VERSION)/test/res.txt`. If all the elements of the solution are 1, then the result is correct. The result on SGI Altix 3700 is shown below. The last two digits vary depending on the environment.

Table 7: Major TARGET options (see `configure` for complete list)

<target>	Configure scripts
cray_xt3	<code>./configure CC=cc FC=ftn CFLAGS="-O3 -B -fastsse -tp k8-64" FCFLAGS="-O3 -fastsse -tp k8-64 -Mpreprocess" FCLDFLAGS="-Mnomain" ac_cv_sizeof_void_p=8 cross_compiling=yes --enable-mpi ax_f77_mangling="lower case, no underscore, extra underscore"</code>
fujitsu_pq	<code>./configure CC=fcc FC=frt ac_cv_sizeof_void_p=8 CFLAGS="-O3 -Kfast,ocl,preex" FFLAGS="-O3 -Kfast,ocl,preex -Cpp" FCFLAGS="-O3 -Kfast,ocl,preex -Cpp -Am" ax_f77_mangling="lower case, underscore, no extra underscore"</code>
hitachi	<code>./configure CC=cc FC=f90 FCLDFLAGS="-lf90s" ac_cv_sizeof_void_p=8 CFLAGS="-Os -noparallel" FCFLAGS="-Oss -noparallel" ax_f77_mangling="lower case, underscore, no extra underscore"</code>
ibm_bgl	<code>./configure CC=blrts_xlc FC=blrts_xlf90 CFLAGS="-O3 -qarch=440d -qtune=440 -qstrict -I/bgl/BlueLight/ppcfloor/bglsys/include" FFFLAGS="-O3 -qarch=440d -qtune=440 -qsuffix=cpp=F -qfixed=72 -w -I/bgl/BlueLight/ppcfloor/bglsys/include" FCFLAGS="-O3 -qarch=440d -qtune=440 -qsuffix=cpp=F90 -w -I/bgl/BlueLight/ppcfloor/bglsys/include" ac_cv_sizeof_void_p=4 cross_compiling=yes --enable-mpi ax_f77_mangling="lower case, no underscore, no extra underscore"</code>
nec_es	<code>./configure CC=esmpic++ FC=esmpif90 AR=esar RANLIB=true ac_cv_sizeof_void_p=8 ax_vector_machine=yes cross_compiling=yes --enable-mpi --enable-omp ax_f77_mangling="lower case, no underscore, extra underscore"</code>
nec_sx9_cross	<code>./configure CC=sxmpic++ FC=sxmpif90 AR=sxar RANLIB=true ac_cv_sizeof_void_p=8 ax_vector_machine=yes cross_compiling=yes ax_f77_mangling="lower case, no underscore, extra underscore"</code>

default

```
100 x 100 matrix  460 entries
Initial vector x = 0
PRECISION : DOUBLE
SOLVER    : BiCG 2
PRECON    : None
STORAGE   : CRS
lis_solve is normal end
```

```
BiCG: iter      = 15 iter_double = 15 iter_quad = 0
BiCG: times     = 5.178690e-03
BiCG: p_times   = 1.277685e-03 (p_c = 1.254797e-03 p_i = 2.288818e-05 )
BiCG: i_times   = 3.901005e-03
BiCG: Residual  = 6.327297e-15
```

--enable-omp

```
Max Procs   = 32
Max Threads = 2
100 x 100 matrix  460 entries
Initial vector x = 0
PRECISION : DOUBLE
SOLVER    : BiCG 2
PRECON    : None
STORAGE   : CRS
lis_solve is normal end
```

```
BiCG: iter      = 15 iter_double = 15 iter_quad = 0
BiCG: times     = 8.960009e-03
BiCG: p_times   = 2.297878e-03 (p_c = 2.072096e-03 p_i = 2.257824e-04 )
BiCG: i_times   = 6.662130e-03
BiCG: Residual  = 6.221213e-15
```

--enable-mpi

```
100 x 100 matrix  460 entries
Initial vector x = 0
PRECISION : DOUBLE
SOLVER    : BiCG 2
PRECON    : None
STORAGE   : CRS
lis_solve is normal end
```

```
BiCG: iter      = 15 iter_double = 15 iter_quad = 0
BiCG: times     = 2.911400e-03
BiCG: p_times   = 1.560780e-04 (p_c = 1.459997e-04 p_i = 1.007831e-05 )
BiCG: i_times   = 2.755322e-03
BiCG: Residual  = 6.221213e-15
```

## 2.6 Installing

In the `lis-($VERSION)` directory, enter as follows:

```
>make install
```

This copies files as follows:

```
$(INSTALLDIR)
+include
|   +lis.h lisf.h
+lib
    +liblis.a
```

`lis.h` and `lisf.h` are the header files required for C and Fortran, respectively. `liblis.a` is the library file.

## 2.7 Test Programs

### 2.7.1 test1

Usage: `test1 matrix_filename rhs_setting solution_filename residual_filename [options]`

This command reads matrix data from `matrix_filename` and solves the linear equation  $Ax = b$  with the solver specified by `options`. It also writes the solution to `solution_filename` and the residual history to `residual_filename`. The MatrixMarket format is supported. One of the following values can be used for `rhs_setting`:

0	use the right hand side vector $b$ , including the matrix file
1	use $b = (1, \dots, 1)^T$
2	use $b = A \times (1, \dots, 1)^T$
<code>rhs_filename</code>	right hand side vector filename

The PLAIN and MM formats are supported for `rhs_filename`. `test1f.f` is the Fortran version of `test1.c`.

### 2.7.2 test2

Usage: `test2 m n matrix_type solution_filename residual_filename [options]`

This command solves a discretized Poisson equation  $Ax = b$  using the five point central difference scheme, with the coefficient matrix in the storage format specified by `matrix_type` and the solver specified by `options`. It also writes the solution to `solution_filename` and the residual history to `residual_filename`. The right hand side vector is set to make all the elements for the solution to be 1. The values `m` and `n` represent the numbers of lattice points in the vertical and horizontal directions.

### 2.7.3 test3

This is a program for solving a linear equation  $Ax = b$  with a specified solver and a preconditioner, where  $A$  is a tridiagonal matrix

$$\begin{pmatrix} 2 & 1 & & & \\ 1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2 & 1 \\ & & & 1 & 2 \end{pmatrix}$$

of size  $12 \times 12$ . The right hand side vector  $b$  is set to make all the elements of the solution  $x$  to be 1. `test3f.f` is the Fortran version of `test3.c`.

### 2.7.4 test4

Usage: `test4 n gamma [options]`

This command solves a linear equation  $Ax = b$ , where  $A$  is a Toeplitz matrix

$$\begin{pmatrix} 2 & 1 & & & \\ 0 & 2 & 1 & & \\ \gamma & 0 & 2 & 1 & \\ & \ddots & \ddots & \ddots & \ddots \\ & & \gamma & 0 & 2 & 1 \\ & & & \gamma & 0 & 2 \end{pmatrix}$$

with the solver specified by `options`. Note that the right hand vector is set to make all the elements of the solution to be 1. The value `n` is the dimension of the matrix  $A$ . The value `gamma` is  $\gamma$ .

### 2.7.5 etest1

Usage: `etest1 matrix_filename solution_filename residual_filename [options]`

This command reads the matrix data from `matrix_filename` and solves an eigenvalue problem  $Ax = \lambda x$  with the solver specified by `options`. It also writes the extreme eigenvector to `solution_filename` and the residual history to `residual_filename`. The MatrixMarket format is supported. `etest1f.f` is the Fortran version of `etest1.c`.

### 2.7.6 etest2

Usage: `etest2 m n matrix_type solution_filename residual_filename [options]`

This command solves an eigenvalue problem  $Ax = \lambda x$ , where the coefficient matrix  $A$  is derived from a discretized Poisson equation using the five point central difference scheme, with the coefficient matrix in the storage format specified by `matrix_type` and the solver specified by `options`. It also writes the extreme eigenvector to `solution_filename` and the residual history to `residual_filename`. The values `m` and `n` represent the numbers of lattice points in the vertical and horizontal directions.

### 2.7.7 etest3

This is a program for solving an eigenvalue problem  $Ax = \lambda x$  with a specified solver, where  $A$  is a tridiagonal matrix

$$\begin{pmatrix} 2 & 1 & & & \\ 1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2 & 1 \\ & & & 1 & 2 \end{pmatrix}$$

of size  $12 \times 12$ . `etest3f.f` is the Fortran version of `etest3.c`.

### 2.7.8 etest4

Usage: `etest4 matrix_filename tol [options]`

This command reads matrix data from `matrix_filename` and compute condition number with convergence criterion `tol`.

### 2.7.9 spmvtest1

Usage: `spmvtest1 m n iter`

This command computes the multiply of a sparse matrix, derived from a discretized Poisson equation using the five point central difference scheme with available matrix storage formats, and a vector  $(1, \dots, 1)^T$ . The values `m` and `n` represent the numbers of lattice points in the vertical and horizontal directions. Performance is measured twice as the average of `iter` iterations.

### 2.7.10 spmvtest2

Usage: `spmvtest2 matrix_filename_list iter`

This command reads matrix data from the files listed in `matrix_filename_list`, and computes the multiplies of the matrices with available matrix storage formats and a vector  $(1, \dots, 1)^T$ . Performance is measured twice as the average of `iter` iterations.

### 2.7.11 `spmvtest3`

Usage: `spmvtest3 matrix_filename matrix_type iter`

This command reads matrix data from `matrix_filename` and compute the multiply of the matrix with `matrix_type` and a vector  $(1, \dots, 1)^T$ . Performance is measured twice as the average of `iter` iterations.

## 2.8 Restrictions

The current version has the following restrictions:

- Preconditioners
  - If a preconditioner other than the Jacobi or SSOR is selected and the matrix  $A$  is not in the CRS format, a new matrix is created in the CRS format for preconditioning.
- Quadruple precision operations
  - Jacobi, Gauss-Seidel, SOR, and IDR(s) methods are not supported.
  - Eigensolvers are not supported.
  - Jacobi, Gauss-Seidel and SOR are not supported in the hybrid preconditioner.
  - I+S and SA-AMG preconditioners are not supported.
- Matrix storage formats
  - In the MPI environment, CRS is the only accepted format for user defined arrays.



### 3 Basic Operations

This section describes how to use the library. Writing programs to solve the linear equation

$$Ax = b$$

or the eigenvalue problem

$$Ax = \lambda x$$

requires the following statements:

- Initialization
- Matrix creation
- Vector creation
- Solver creation
- Value assignment for matrix and vector
- Solver assignment for linear equation or eigenvalue problem
- Solver execution
- Finalization

In addition, each program must include one of the following `include` statements:

- C `#include "lis.h"`
- Fortran `#include "lisf.h"`

When Lis is installed in `$(INSTALLDIR)`, `lis.h` and `lisf.h` are located in `$(INSTALLDIR)/include`.

### 3.1 Initialization and Finalization

The initialization and finalization of the execution environment must be called at the beginning and the end of the program as follows:

```
C
1: #include "lis.h"
2: int main(int argc, char* argv[])
3: {
4:     lis_initialize(&argc, &argv);
5:     ...
6:     lis_finalize();
7: }
```

```
Fortran
1: #include "lisf.h"
2:     call lis_initialize(ierr)
3:     ...
4:     call lis_finalize(ierr)
```

#### Initialization

To call initialization, the following functions are used:

- C            `lis_initialize(int* argc, char** argv())`
- Fortran subroutine `lis_initialize(integer ierr)`

This function calls initialization of MPI, and specifies options on the command line.

#### Finalization

To call finalization, the following functions are used:

- C            `int lis_finalize()`
- Fortran subroutine `lis_finalize(integer ierr)`

### 3.2 Vector Operations

Assume that the size of the vector  $v$  is `global_n`, and the size of each partial vector divided among `nprocs` processors is `local_n`. If `global_n` is divisible, then `local_n = global_n / nprocs`. For example, when the vector  $v$  is divided between two processors, as shown by Equation (3.1), `global_n` and `local_n` are 4 and 2, respectively.

$$v = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} \text{PE0} \\ \text{PE1} \end{matrix} \quad (3.1)$$

In the case of creating the vector  $v$  in Equation (3.1), the sequential and OpenMP versions create the vector  $v$  itself, while the MPI version creates partial vectors divided among a given number of processors.

Programs to create the vector  $v$  are as follows, where the number of processors for the MPI version is assumed to be two:

C(Sequential, OpenMP)

```
1: int          i,n;
2: LIS_VECTOR   v;
3: n = 4;
4: lis_vector_create(0,&v);
5: lis_vector_set_size(v,0,n); /* or lis_vector_set_size(v,n,0); */
6:
7: for(i=0;i<n;i++)
8: {
9:     lis_vector_set_value(LIS_INS_VALUE,i,(double)i,v);
10: }
```

C(MPI)

```
1: int          i,n,is,ie;                /*or int  i,ln,is,ie;                */
2: LIS_VECTOR   v;
3: n = 4;                                  /*  ln = 2;                */
4: lis_vector_create(MPI_COMM_WORLD,&v);
5: lis_vector_set_size(v,0,n);             /*  lis_vector_set_size(v,ln,0); */
6: lis_vector_get_range(v,&is,&ie);
7: for(i=is;i<ie;i++)
8: {
9:     lis_vector_set_value(LIS_INS_VALUE,i,(double)i,v);
10: }
```

Fortran(Sequential, OpenMP)

```
1: integer      i,n
2: LIS_VECTOR   v
3: n = 4
4: call lis_vector_create(0,v,ierr)
5: call lis_vector_set_size(v,0,n,ierr)
6:
7: do i=1,n
9:     call lis_vector_set_value(LIS_INS_VALUE,i,DBLE(i),v,ierr)
10: enddo
```

Fortran(MPI)

```
1: integer      i,n,is,ie
2: LIS_VECTOR   v
3: n = 4
4: call lis_vector_create(MPI_COMM_WORLD,v,ierr)
5: call lis_vector_set_size(v,0,n,ierr)
6: call lis_vector_get_range(v,is,ie,ierr)
7: do i=is,ie-1
8:     call lis_vector_set_value(LIS_INS_VALUE,i,DBLE(i),v,ierr);
9: enddo
```

## Declaring variable

As the second line shows, the declaration is stated as follows:

```
LIS_VECTOR    v;
```

## Creating Vector

To create a vector *v*, the following functions are used:

- C `int lis_vector_create(LIS_Comm comm, LIS_VECTOR *v)`
- Fortran subroutine `lis_vector_create(LIS_Comm comm, LIS_VECTOR v, integer ierr)`

For the example program above, `comm` must be replaced with the MPI communicator. For the sequential and OpenMP versions, the value for `comm` is ignored.

## Assigning Vector Size

To assign the size of a vector *v*, the following functions are used:

- C `int lis_vector_set_size(LIS_VECTOR v, int local_n, int global_n)`
- Fortran subroutine `lis_vector_create(integer local_n, integer global_n, LIS_Comm comm, LIS_VECTOR v, integer ierr)`

Either `local_n` or `global_n` must be provided. This function creates a vector in one of the following ways: Creates partial vectors of size `local_n` if `local_n` is given, or create partial vectors of size `global_n`, divided among a given number of processors, if `global_n` is given.

In the case of the sequential and OpenMP versions, `local_n = global_n`. This means that both `lis_vector_set_size(v,n,0)` and `lis_vector_set_size(v,0,n)` create a vector of size *n*.

For the MPI version, `lis_vector_set_size(v,n,0)` creates a partial vector of size  $n_p$  on the processor *p*. On the other hand, `lis_vector_set_size(v,0,n)` creates a partial vector of size  $m_p$  on the processor *p*. The value for  $m_p$  is determined by the library.

## Assigning Element

To assign an element to the *i*-th row of the vector *v*, the following functions are used:

- C `int lis_vector_set(int flag, int i, LIS_SCALAR value, LIS_VECTOR v)`
- Fortran subroutine `lis_vector_set_value(int flag, int i, LIS_SCALAR value, LIS_VECTOR v, integer ierr)`

For the MPI version, the *i*-th row of the global vector must be specified, rather than the *i*-th row of the partial vector. Either

`LIS_INS.VALUE : v[i] = value`, or

`LIS_ADD.VALUE : v[i] = v[i] + value`

must be provided for `flag`.

## Duplicating Vector

To create a vector which has the same information as for an existing vector, the following functions are used:

- C `int lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR *vout)`
- Fortran subroutine `lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR vout, integer ierr)`

This function does not copy the elements of the vector. To copy the elements as well, the following functions must be added after the above functions:

- C `int lis_vector_copy(LIS_VECTOR vsrc, LIS_VECTOR vdst)`
- Fortran subroutine `lis_vector_copy(LIS_VECTOR vsrc, LIS_VECTOR vdst, integer ierr)`

### Destroying Vector

To destroy a vector, the following functions are used:

- C `int lis_vector_destroy(LIS_VECTOR v)`
- Fortran subroutine `lis_vector_destroy(LIS_VECTOR v, integer ierr)`

## 3.3 Matrix Operations

Assume that the size of the matrix  $A$  is  $\text{global\_n} \times \text{global\_n}$ , and that the size of each row block of the matrix  $A$  divided among  $\text{nprocs}$  processors is  $\text{local\_n} \times \text{global\_n}$ . If  $\text{global\_n}$  is divisible, then  $\text{local\_n} = \text{global\_n} / \text{nprocs}$ . For example, when the row block of the matrix  $A$  is divided between two processors, as shown by Equation (3.2),  $\text{global\_n}$  and  $\text{local\_n}$  are 4 and 2, respectively.

$$A = \left( \begin{array}{ccc} 2 & 1 & \\ 1 & 2 & 1 \\ & 1 & 2 & 1 \\ & & 1 & 2 \end{array} \right) \begin{array}{l} \text{PE0} \\ \text{PE1} \end{array} \quad (3.2)$$

A matrix for a specific storage format can be created in one of the following three ways:

1. The library prepares the arrays required by the specified storage type.
2. The user prepares the arrays required by the specified storage type.
3. Matrix and vector data are read from a file.

### Approach 1: The library prepares the arrays required by the specific storage type

In the case of creating the matrix  $A$  in Equation (3.2) in the CRS format, the sequential and OpenMP versions create the matrix  $A$  itself, and the MPI version creates on each processor a partial matrix divided among given number of processors.

Programs to create the matrix  $A$  in the CRS format are as follows, where the number of processors for the MPI version is assumed to be two:

C(Sequential, OpenMP)

```

1: int          i,n;
2: LIS_MATRIX   A;
3: n = 4;
4: lis_matrix_create(0,&A);
5: lis_matrix_set_size(A,0,n); /* or lis_matrix_set_size(A,n,0); */
6: for(i=0;i<n;i++) {
7:     if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
8:     if( i<n-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);
9:     lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
10: }
11: lis_matrix_set_type(A,LIS_MATRIX_CRS);
12: lis_matrix_assemble(A);

```

#### C(MPI)

```
1: int          i,n,gn,is,ie;
2: LIS_MATRIX   A;
3: gn = 4;                      /* or n=2 */
4: lis_matrix_create(MPI_COMM_WORLD,&A);
5: lis_matrix_set_size(A,0,gn);  /* lis_matrix_set_size(A,n,0); */
6: lis_matrix_get_size(A,&n,&gn);
7: lis_matrix_get_range(A,&is,&ie);
8: for(i=is;i<ie;i++) {
9:     if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
10:    if( i<gn-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);
11:    lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
12: }
13: lis_matrix_set_type(A,LIS_MATRIX_CRD);
14: lis_matrix_assemble(A);
```

#### Fortran(Sequential, OpenMP)

```
1: integer      i,n
2: LIS_MATRIX   A
3: n = 4
4: call lis_matrix_create(0,A,ierr)
5: call lis_matrix_set_size(A,0,n,ierr)
6: do i=1,n
7:     if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0d0,A,ierr)
8:     if( i<n ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0d0,A,ierr)
9:     call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
10: enddo
11: call lis_matrix_set_type(A,LIS_MATRIX_CRD,ierr)
12: call lis_matrix_assemble(A,ierr)
```

#### Fortran(MPI)

```
1: integer      i,n,gn,is,ie
2: LIS_MATRIX   A
3: gn = 4
4: call lis_matrix_create(MPI_COMM_WORLD,A,ierr)
5: call lis_matrix_set_size(A,0,gn,ierr)
6: call lis_matrix_get_size(A,n,gn,ierr)
7: call lis_matrix_get_range(A,is,ie,ierr)
8: do i=is,ie-1
9:     if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0d0,A,ierr)
10:    if( i<gn ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0d0,A,ierr)
11:    call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
12: enddo
13: call lis_matrix_set_type(A,LIS_MATRIX_CRD,ierr)
14: call lis_matrix_assemble(A,ierr)
```

### Declaring Variable

As the second line shows, the declaration is stated as follows:

```
LIS_MATRIX   A;
```

### Creating Matrix

To create the matrix A, the following functions are used:

- C            `int lis_matrix_create(LIS_Comm comm, LIS_MATRIX *A)`
- Fortran subroutine `lis_matrix_create(LIS_Comm comm, LIS_MATRIX A, integer ierr)`

`comm` must be replaced with the MPI communicator. For the sequential and OpenMP versions, the value for `comm` is ignored.

### Assigning Matrix Size

To assign a size to the matrix A, the following functions are used:

- C            `int lis_matrix_set_size(LIS_MATRIX A, int local_n, int global_n)`
- Fortran subroutine `lis_matrix_set_size(LIS_MATRIX A, integer local_n, integer global_n, integer ierr)`

Either `local_n` or `global_n` must be provided. This function creates matrices in one of the following two ways: Create partial matrices of size `local_n` x `N` if `local_n` is given, or creating partial matrices of size `global_n` x `global_n` divided among a given number of processors, if `global_n` is given. `N` represents the total sum of `local_n`.

In the case of the sequential and OpenMP versions, `local_n = global_n`. This means that both `lis_matrix_set_size(A,n,0)` and `lis_matrix_set_size(A,0,n)` create a matrix of `n` x `n`.

For the MPI version, `lis_matrix_set_size(A,n,0)` creates a partial matrix of size  $n_p \times N$  on the processor  $p$ , where  $N$  is the total sum of  $n_p$ . On the other hand, `lis_matrix_set_size(A,0,n)` creates a partial matrix of size  $m_p \times n$  on the processor  $p$ , where  $m_p$  is the number of the partial matrix, which is determined by the library.

### Assigning Element

To assign an element to the cell at the  $i$ -th row and the  $j$ -th column of the matrix A, the following functions are used:

- C            `int lis_matrix_set_value(int flag, int i, int j, LIS_SCALAR value, LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_value(integer flag, integer i, integer j, LIS_SCALAR value, LIS_MATRIX A, integer ierr)`

For the MPI version, the  $i$ -th row and the  $j$ -th column of the global matrix must be specified, rather than the  $i$ -th row and the  $j$ -th column of the partial matrix. Either

`LIS_INS_VALUE` :  $A(i,j) = \text{value}$ , or

`LIS_ADD_VALUE` :  $A(i,j) = A(i,j) + \text{value}$

must be provided for `flag`.

### Assigning Storage Format

To assign a storage format to the matrix A, the following functions are used:

- C            `int lis_matrix_set_type(LIS_MATRIX A, int matrix_type)`
- Fortran subroutine `lis_matrix_set_type(LIS_MATRIX A, int matrix_type, integer ierr)`

`matrix_type` of A is `LIS_MATRIX_CRS` when the matrix is created. The following storage formats are accepted:

Storage formats		matrix_type
Compressed Row Storage	(CRS)	{LIS_MATRIX_CRS 1}
Compressed Column Storage	(CCS)	{LIS_MATRIX_CCS 2}
Modified Compressed Sparse Row	(MSR)	{LIS_MATRIX_MSR 3}
Diagonal	(DIA)	{LIS_MATRIX_DIA 4}
Ellpack-Itpack generalized diagonal	(ELL)	{LIS_MATRIX_ELL 5}
Jagged Diagonal	(JDS)	{LIS_MATRIX_JDS 6}
Block Sparse Row	(BSR)	{LIS_MATRIX_BSR 7}
Block Sparse Column	(BSC)	{LIS_MATRIX_BSC 8}
Variable Block Row	(VBR)	{LIS_MATRIX_VBR 9}
Dense	(DNS)	{LIS_MATRIX_DNS 10}
Coordinate	(COO)	{LIS_MATRIX_COO 11}

### Assembling Matrix

After assigning elements, the following function must be used:

- C `int lis_matrix_assemble(LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_assemble(LIS_MATRIX A, integer ierr)`

`lis_matrix_assemble` is assembled to the storage format specified with `lis_matrix_set_type`.

### Destroying Matrix

To destroy a matrix, the following functions are used:

- C `int lis_matrix_destroy(LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_destroy(LIS_MATRIX A, integer ierr)`

### Approach 2: The user prepares the arrays required by the specified storage type

In the case of creating the matrix  $A$  in Equation (3.2) in the CRS format, the sequential and OpenMP versions creates the matrix  $A$  itself, and the MPI version creates on each processor a partial matrix divided among a given number of processors.

Programs to create the matrix  $A$  in the CRS format are as follows, where the number of processors for the MPI version is assumed to be two:



C(Sequential, OpenMP)

```

1: int          i,k,n,nnz;
2: int          *ptr,*index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; nnz = 10; k = 0;
6: lis_matrix_malloc_crs(n,nnz,&ptr,&index,&value);
7: lis_matrix_create(0,&A);
8: lis_matrix_set_size(A,0,n); /* or lis_matrix_set_size(A,n,0); */
9:
10: for(i=0;i<n;i++)
11: {
12:     if( i>0 ) {index[k] = i-1; value[k] = 1; k++;}
13:     index[k] = i; value[k] = 2; k++;
14:     if( i<n-1 ) {index[k] = i+1; value[k] = 1; k++;}
15:     ptr[i+1] = k;
16: }
17: ptr[0] = 0;
18: lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);

```

C(MPI)

```

1: int          i,k,n,nnz,is,ie;
2: int          *ptr,*index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 2; nnz = 5; k = 0;
6: lis_matrix_malloc_crs(n,nnz,&ptr,&index,&value);
7: lis_matrix_create(MPI_COMM_WORLD,&A);
8: lis_matrix_set_size(A,n,0);
9: lis_matrix_get_range(A,&is,&ie);
10: for(i=is;i<ie;i++)
11: {
12:     if( i>0 ) {index[k] = i-1; value[k] = 1; k++;}
13:     index[k] = i; value[k] = 2; k++;
14:     if( i<n-1 ) {index[k] = i+1; value[k] = 1; k++;}
15:     ptr[i-is+1] = k;
16: }
17: ptr[0] = 0;
18: lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);

```

### Associating Array

To associate the arrays required by the CRS format created by the user with the matrix A, the following functions are used:

- C `int lis_matrix_set_crs(int nnz, int row[], int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_crs(integer nnz, integer row(), integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

### Approach 3: Matrix and vector data are read from a file

Programs to read the matrix  $A$  in Equation (3.2) in CRS format and vector  $b$  in Equation (3.1) from a file are as follows:

C(Sequential, OpenMP, MPI)

```
1: LIS_MATRIX    A;
2: LIS_VECTOR    b,x;
3: lis_matrix_create(LIS_COMM_WORLD,&A);
4: lis_vector_create(LIS_COMM_WORLD,&b);
5: lis_vector_create(LIS_COMM_WORLD,&x);
6: lis_matrix_set_type(A,LIS_MATRIX_CRS);
7: lis_input(A,b,x,"matvec.mtx");
```

Fortran(Sequential, OpenMP, MPI)

```
1: LIS_MATRIX    A
2: LIS_VECTOR    b,x
3: call lis_matrix_create(LIS_COMM_WORLD,A,ierr)
4: call lis_vector_create(LIS_COMM_WORLD,b,ierr)
5: call lis_vector_create(LIS_COMM_WORLD,x,ierr)
6: call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
7: call lis_input(A,b,x,'matvec.mtx',ierr)
```

The content of the target file `matvec.mtx` is as follows:

```
%%MatrixMarket matrix coordinate real general
4 4 10 1 0
1 2 1.0e+00
1 1 2.0e+00
2 3 1.0e+00
2 1 1.0e+00
2 2 2.0e+00
3 4 1.0e+00
3 2 1.0e+00
3 3 2.0e+00
4 4 2.0e+00
4 3 1.0e+00
1 0.0e+00
2 1.0e+00
3 2.0e+00
4 3.0e+00
```

### Reading from File

To read the data for the matrix  $A$  from a file, the following functions are used:

- C `int lis_input_matrix(LIS_MATRIX A, char *filename)`
- Fortran subroutine `lis_input(LIS_MATRIX A, character filename, integer ierr)`

`filename` must be replaced with the path to the target file. The following file formats are supported:

- MatrixMarket format
- Harwell-Boeing format
- Lis format (original format)

To read the data for the matrix  $A$  and vectors  $b$  and  $x$  from a file, the following functions are used:

- C `int lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, char *filename)`
- Fortran subroutine `lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, character filename, integer ierr)`

filename must be replaced with the path to the target file. The following file formats are supported:

- MatrixMarket format (extended to allow vector data to be read)
- Harwell-Boeing format
- Lis format (original format)

### 3.4 Solving Linear Equation

The program to solve the linear equation  $Ax = b$  with a specified solver is as follows:

C(Sequential, OpenMP, MPI)

```
1: LIS_MATRIX A;
2: LIS_VECTOR b,x;
3: LIS_SOLVER solver;
4:
5: /* Create matrix and vector */
6:
7: lis_solver_create(&solver);
8: lis_solver_set_option("-i bicg -p none",solver);
9: lis_solver_set_option("-tol 1.0e-12",solver);
10: lis_solver(A,b,x,solver);
```

Fortran(Sequential, OpenMP, MPI)

```
1: LIS_MATRIX A
2: LIS_VECTOR b,x
3: LIS_SOLVER solver
4:
5: /* Create matrix and vector */
6:
7: call lis_solver_create(solver,ierr)
8: call lis_solver_set_option('-i bicg -p none',solver,ierr)
9: call lis_solver_set_option('-tol 1.0e-12',solver,ierr)
10: call lis_solver(A,b,x,solver,ierr)
```

#### Creating Solver

To create a solver, the following functions are used:

- C `int lis_solver_create(LIS_SOLVER *solver)`
- Fortran subroutine `lis_solver_create(LIS_SOLVER solver, integer ierr)`

#### Specifying Option

To specify options, the following functions are used:

- C `int lis_solver_set_option(char *text, LIS_SOLVER solver)`
- Fortran subroutine `lis_solver_set_option(character text, LIS_SOLVER solver, integer ierr)`

or

- C `int lis_solver_set_optionC(LIS_SOLVER solver)`
- Fortran subroutine `lis_solver_set_optionC(LIS_SOLVER solver, integer ierr)`

`lis_solver_set_optionC` is a function which sets the options specified on the command line, and pass them to `solver` when the user's program is run.

The table below shows the available command line options, where `-i {cg|1}` means `-i cg` or `-i 1` and `-maxiter [1000]` indicates that `-maxiter` defaults to 1,000.

**Specifying Linear Equation Solver (Default: `-i bicg`)**

Method	Option	Auxiliary Option	
CG	<code>-i {cg 1}</code>		
BiCG	<code>-i {bicg 2}</code>		
CGS	<code>-i {cgs 3}</code>		
BiCGSTAB	<code>-i {bicgstab 4}</code>		
BiCGSTAB(l)	<code>-i {bicgstabl 5}</code>	<code>-ell [2]</code>	Value for l
GPBiCG	<code>-i {gpbicg 6}</code>		
TFQMR	<code>-i {tfqmr 7}</code>		
Orthomin(m)	<code>-i {orthomin 8}</code>	<code>-restart [40]</code>	Value for restart m
GMRES(m)	<code>-i {gmres 9}</code>	<code>-restart [40]</code>	Value for restart m
Jacobi	<code>-i {jacobi 10}</code>		
Gauss-Seidel	<code>-i {gs 11}</code>		
SOR	<code>-i {sor 12}</code>	<code>-omega [1.9]</code>	Value for relaxation coefficient $\omega$ ( $0 < \omega < 2$ )
BiCGSafe	<code>-i {bicgsafe 13}</code>		
CR	<code>-i {cr 14}</code>		
BiCR	<code>-i {bicr 15}</code>		
CRS	<code>-i {crs 16}</code>		
BiCRSTAB	<code>-i {bicrstab 17}</code>		
GPBiCR	<code>-i {gpbicr 18}</code>		
BiCRSafe	<code>-i {bicrsafe 19}</code>		
FGMRES(m)	<code>-i {fgmres 20}</code>	<code>-restart [40]</code>	Value for restart m
IDR(s)	<code>-i {idrs 21}</code>	<code>-restart [40]</code>	Value for restart s
MINRES	<code>-i {minres 22}</code>		

### Specifying Preconditioner (Default: -p none)

Preconditioner	Option	Auxiliary Option	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	Fill level $k$
SSOR	-p {ssor 3}	-ssor_w [1.0]	Relaxation coefficient $\omega$ ( $0 < \omega < 2$ )
hybrid	-p {hybrid 4}	-hybrid_i [sor]	Linear equation solver
		-hybrid_maxiter [25]	Maximum number of iterations
		-hybrid_tol [1.0e-3]	Convergence criteria
		-hybrid_w [1.5]	Relaxation coefficient $\omega$ for the SOR method ( $0 < \omega < 2$ )
		-hybrid_ell [2]	Value for $l$ of the BiCGSTAB(l) method
I+S	-p {is 5}	-hybrid_restart [40]	Restart values for GMRES and Orthomin
		-is_alpha [1.0]	Parameter $\alpha$ for preconditioner of a $I + \alpha S^{(m)}$ type
		-is_m [3]	Parameter $m$ for preconditioner of a $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05]	Drop criteria
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Selection of unsymmetric version (Matrix structure must be symmetric.)
		-saamg_theta [0.05 0.12]	Drop criteria $a_{ij}^2 \leq \theta^2  a_{ii}   a_{jj} $ (symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	Drop criteria
		-iluc_rate [5.0]	Ratio of maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05]	Drop criteria
		-ilut_rate [5.0]	Ratio of maximum fill-in
additive Schwarz	-adds true	-adds_iter [1]	Number of iterations

### Other Options

Option	
-maxiter [1000]	Maximum number of iterations
-tol [1.0e-12]	Convergence criteria
-print [0]	Display of the residual
	-print {none 0} None
	-print {mem 1} Saves the residual history in memory
	-print {out 2} Displays the residual history
	-print {all 3} Saves the residual history and displays it on the screen
-scale [0]	Selection of scaling. The result will overwrite the original matrix and vectors
	-scale {none 0} No scaling
	-scale {jacobi 1} Jacobi scaling $D^{-1}Ax = D^{-1}b$ $D$ represents the diagonal of $A = (a_{ij})$
	-scale {symm_diag 2} Diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$ $D^{-1/2}$ represents a diagonal matrix with $1/\sqrt{a_{ii}}$ as diagonal
-initx_zeros [true]	Behavior of the initial vector $x_0$
	-initx_zeros {false 0} Given values
	-initx_zeros {true 1} All elements are set to 0.
-omp_num_threads [t]	Number of threads $t$ represents the maximum number of threads

Precision (Default: -precision double)		
Precision	Option	Auxiliary Option
DOUBLE	-precision {double 0}	
QUAD	-precision {quad 1}	

### Solving Linear Equation

To solve the linear equation  $Ax = b$ , the following functions are used:

- C `int lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver)`
- Fortran subroutine `lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver, integer ierr)`

## 3.5 Solving Eigenvalue Problem

The program to solve the eigenvalue problem  $Ax = \lambda x$  with a specified solver is as follows:

C(Sequential, OpenMP, MPI)

```
1: LIS_MATRIX A;
2: LIS_VECTOR x;
3: LIS_REAL eval;
4: LIS_ESOLVER esolver;
5:
6: /* Create matrix and vector */
7:
8: lis_esolver_create(&esolver);
9: lis_esolver_set_option("-e ii -i bicg -p none",esolver);
10: lis_esolver_set_option("-etol 1.0e-12 -tol 1.0e-12",esolver);
11: lis_solve(A,x,eval,esolver);
```

Fortran(Sequential, OpenMP, MPI)

```
1: LIS_MATRIX A
2: LIS_VECTOR x
3: LIS_REAL eval
4: LIS_ESOLVER esolver
5:
6: /* Create matrix and vector */
7:
8: call lis_esolver_create(esolver,ierr)
9: call lis_esolver_set_option('-e ii -i bicg -p none',esolver,ierr)
10: call lis_esolver_set_option('-etol 1.0e-12 -tol 1.0e-12',esolver,ierr)
11: call lis_solve(A,x,eval,esolver,ierr)
```

### Creating Eigensolver

To create an eigensolver, the following functions are used:

- C `int lis_esolver_create(LIS_ESOLVER *esolver)`
- Fortran subroutine `lis_esolver_create(LIS_ESOLVER esolver, integer ierr)`

### Specifying Option

To specify options, the following functions are used:

- C `int lis_esolver_set_option(char *text, LIS_ESOLVER esolver)`

- Fortran subroutine `lis_esolver_set_option(character text, LIS_ESOLVER esolver, integer ierr)`

or

- C `int lis_esolver_set_optionC(LIS_ESOLVER esolver)`
- Fortran subroutine `lis_esolver_set_optionC(LIS_ESOLVER esolver, integer ierr)`

`lis_esolver_set_optionC` is a function which sets the options specified on the command line, and pass them to `esolver` when the user's program is run.

The table below shows the available command line options, where `-e {pi|1}` means `-e pi` or `-e 1` and `-emaxiter [1000]` indicates that `-emaxiter` defaults to 1,000.

**Specifying Eigensolver** (Default: `-i bicg`)

Method	Option	Auxiliary Option
Power Iteration	<code>-e {pi 1}</code>	
Inverse Iteration	<code>-e {ii 2}</code>	<code>-i [bicg]</code> Linear equation solver
Approximate Inverse Iteration	<code>-e {aii 3}</code>	
Conjugate Gradient	<code>-e {cg 4}</code>	
Lanczos Iteration	<code>-e {li 5}</code>	<code>-ss [10]</code> Size of subspace
Subspace Iteration	<code>-e {si 6}</code>	<code>-ss [10]</code> Size of subspace
Conjugate Residual	<code>-e {cr 7}</code>	

**Specifying Preconditioner** (Default: `-p ilu`)

Preconditioner	Option	Auxiliary Option
None	<code>-p {none 0}</code>	
Jacobi	<code>-p {jacobi 1}</code>	
ILU(k)	<code>-p {ilu 2}</code>	<code>-ilu_fill [0]</code> Fill level $k$
SSOR	<code>-p {ssor 3}</code>	<code>-ssor_w [1.0]</code> Relaxation coefficient $\omega$ ( $0 < \omega < 2$ )
hybrid	<code>-p {hybrid 4}</code>	<code>-hybrid_i [sor]</code> Linear equation solver
		<code>-hybrid_maxiter [25]</code> Maximum number of iterations
		<code>-hybrid_tol [1.0e-3]</code> Convergence criteria
		<code>-hybrid_w [1.5]</code> Relaxation coefficient $\omega$ for the SOR method ( $0 < \omega < 2$ )
		<code>-hybrid_ell [2]</code> Value for $l$ of the BiCGSTAB(1) method
		<code>-hybrid_restart [40]</code> Restart values for GMRES and Orthomin
I+S	<code>-p {is 5}</code>	<code>-is_alpha [1.0]</code> Parameter $\alpha$ for preconditioner of a $I + \alpha S^{(m)}$ type
		<code>-is_m [3]</code> Parameter $m$ for preconditioner of a $I + \alpha S^{(m)}$ type
SAINV	<code>-p {sainv 6}</code>	<code>-sainv_drop [0.05]</code> Drop criteria
SA-AMG	<code>-p {saamg 7}</code>	<code>-saamg_unsym [false]</code> Selection of unsymmetric version (Matrix structure must be symmetric.)
		<code>-saamg_theta [0.05 0.12]</code> Drop criteria $a_{ij}^2 \leq \theta^2  a_{ii}   a_{jj} $ (symmetric or unsymmetric)
Crout ILU	<code>-p {iluc 8}</code>	<code>-iluc_drop [0.05]</code> Drop criteria
		<code>-iluc_rate [5.0]</code> Ratio of maximum fill-in
ILUT	<code>-p {ilut 9}</code>	<code>-ilut_drop [0.05]</code> Drop criteria
		<code>-ilut_rate [5.0]</code> Ratio of maximum fill-in
additive Schwarz	<code>-adds true</code>	<code>-adds_iter [1]</code> Number of iterations

## Other Options

Option	
<code>-emaxiter [1000]</code>	Maximum number of iterations
<code>-etol [1.0e-5]</code>	Convergence criteria
<code>-eprint [0]</code>	Display of the residual
	<code>-eprint {none 0}</code> None
	<code>-eprint {mem 1}</code> Saves the residual history in memory
	<code>-eprint {out 2}</code> Displays the residual history
	<code>-eprint {all 3}</code> Saves the residual history and displays it on the screen
<code>-ie [ii]</code>	Inner eigensolver used in Lanczos Iteration or Subspace Iteration
	<code>-ie {pi 1}</code> Power Iteration (Subspace Iteration only)
	<code>-ie {ii 2}</code> Inverse Iteration
	<code>-ie {aii 3}</code> Approximate Inverse Iteration
<code>-shift [0.0]</code>	Amount of shift
<code>-initx_ones [true]</code>	Behavior of the initial vector $x_0$
	<code>-initx_ones {false 0}</code> Given values
	<code>-initx_ones {true 1}</code> All elements are set to 1.
<code>-omp_num_threads [t]</code>	Number of threads
	<code>t</code> represents the maximum number of threads

### Solving Eivenvalue Problem

To solve the eigenvalue problem  $Ax = \lambda x$ , the following functions are used:

- C      `int lis_esolve(LIS_MATRIX A, LIS_VECTOR x, LIS_REAL eval, LIS_ESOLVER esolver)`
- Fortran subroutine `lis_esolve(LIS_MATRIX A, LIS_VECTOR x, LIS_REAL eval, LIS_ESOLVER esolver, integer ierr)`

### 3.6 Sample Programs

This is a program for solving the linear equation  $Ax = b$  with a specified solver to provide an approximate solution for the equation, where the matrix  $A$  is a tridiagonal matrix

$$\begin{pmatrix} 2 & 1 & & & \\ 1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2 & 1 \\ & & & 1 & 2 \end{pmatrix}$$

of size  $12 \times 12$ . The the right hand side vector  $b$  is set to make all the elements of the solution  $x$  is 1.

This sample program is stored in the directory `lis-($VERSION)/test`.



Test program: test3.c

```
1: #include <stdio.h>
2: #include "lis.h"
3: main(int argc, char *argv[])
4: {
5:     int i,n,gn,is,ie,iter;
6:     LIS_MATRIX A;
7:     LIS_VECTOR b,x,u;
8:     LIS_SOLVER solver;
9:     n = 12;
10:    lis_initialize(&argc,&argv);
11:    lis_matrix_create(MPI_COMM_WORLD,&A);
12:    lis_matrix_set_size(A,0,n);
13:    lis_matrix_get_size(A,&n,&gn)
14:    lis_matrix_get_range(A,&is,&ie)
15:    for(i=is;i<ie;i++)
16:    {
17:        if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
18:        if( i<gn-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);
19:        lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
20:    }
21:    lis_matrix_set_type(A,LIS_MATRIX_CRS);
22:    lis_matrix_assemble(A);
23:
24:    lis_vector_duplicate(A,u);
25:    lis_vector_duplicate(A,b);
26:    lis_vector_duplicate(A,x);
27:    lis_vector_set_all(1.0,u);
28:    lis_matvec(A,u,b);
29:
30:    lis_solver_create(&solver);
31:    lis_solver_set_optionC(solver);
32:    lis_solve(A,b,x,solver);
33:    lis_solver_get_iters(solver,&iter);
34:    printf("iter = %d\n",iter);
35:    lis_vector_print(x);
36:    lis_matrix_destroy(A);
37:    lis_vector_destroy(u);
38:    lis_vector_destroy(b);
39:    lis_vector_destroy(x);
40:    lis_solver_destroy(solver);
41:    lis_finalize();
42:    return 0;
43: }
```

Test program: test3f.f

```
1:      implicit none
2:
3: #include "lisf.h"
4:
5:      integer          i,n,gn,is,ie,iter,ierr
6:      LIS_MATRIX       A
7:      LIS_VECTOR       b,x,u
8:      LIS_SOLVER       solver
9:      n = 12
10:     call lis_initialize(ierr)
11:     call lis_matrix_create(LIS_COMM_WORLD,A,ierr)
12:     call lis_matrix_set_size(A,0,n,ierr)
13:     call lis_matrix_get_size(A,n,gn,ierr)
14:     call lis_matrix_get_range(A,is,ie,ierr)
15:     do i=is,ie-1
16:         if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0d0,
17:                                             A,ierr)
18:         if( i<gn ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0d0,
19:                                             A,ierr)
20:         call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
21:     enddo
22:     call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
23:     call lis_matrix_assemble(A,ierr)
24:
25:     call lis_vector_duplicate(A,u,ierr)
26:     call lis_vector_duplicate(A,b,ierr)
27:     call lis_vector_duplicate(A,x,ierr)
28:     call lis_vector_set_all(1.0d0,u,ierr)
29:     call lis_matvec(A,u,b,ierr)
30:
31:     call lis_solver_create(solver,ierr)
32:     call lis_solver_set_optionC(solver,ierr)
33:     call lis_solve(A,b,x,solver,ierr)
34:     call lis_solver_get_iters(solver,iter,ierr)
35:     write(*,*) 'iter = ',iter
36:     call lis_vector_print(x,ierr)
37:     call lis_matrix_destroy(A,ierr)
38:     call lis_vector_destroy(b,ierr)
39:     call lis_vector_destroy(x,ierr)
40:     call lis_vector_destroy(u,ierr)
41:     call lis_solver_destroy(solver,ierr)
42:     call lis_finalize(ierr)
43:
44:     stop
45:     end
```

### 3.7 Compiling and Linking

Provided below is an example of a test program `test3.c` located in the directory `lis-($VERSION)/test`, compiled on SGI Altix 3700 using Intel C/C++ Compiler 8.1 (icc) and Intel Fortran Compiler 8.1 (ifort). Since the library includes Fortran 90 codes when the SA-AMG preconditioner is used, the link is processed with the Fortran 90 compiler.

#### Sequential

##### Compiling

```
>icc -c -I$(INSTALLDIR)/include test3.c
```

##### Linking

```
>icc -o test3 test3.o -llis
```

##### Linking (with SA-AMG)

```
>ifort -nofor_main -o test3 test3.o -llis
```

#### OpenMP

##### Compiling

```
>icc -c -openmp -I$(INSTALLDIR)/include test3.c
```

##### Linking

```
>icc -openmp -o test3 test3.o -llis
```

##### Linking (with SA-AMG)

```
>ifort -nofor_main -openmp -o test3 test3.o -llis
```

#### MPI

##### Compiling

```
>icc -c -DUSE_MPI -I$(INSTALLDIR)/include test3.c
```

##### Linking

```
>icc -o test3 test3.o -llis -lmpi
```

##### Linking (with SA-AMG)

```
>ifort -nofor_main -o test3 test3.o -llis -lmpi
```

#### OpenMP + MPI

##### Compiling

```
>icc -c -openmp -DUSE_MPI -I$(INSTALLDIR)/include test3.c
```

##### Linking

```
>icc -openmp -o test3 test3.o -llis -lmpi
```

##### Linking (with SA-AMG)

```
>ifort -nofor_main -openmp -o test3 test3.o -llis -lmpi
```

Provided below is an example of a test program `test3f.f` located in the directory `lis-($VERSION)/test`, compiled on SGI Altix 3700 using Intel Fortran Compiler 8.1 (ifort). Since an `#include` statement is used in the program, a compiler option `-fpp` is specified to use the pre-processor.

#### Sequential

##### Compiling

```
>ifort -c -fpp -I$(INSTALLDIR)/include test3f.f
```

##### Linking

```
>ifort -o test3 test3.o -llis
```

#### OpenMP

##### Compiling

```
>ifort -c -fpp -openmp -I$(INSTALLDIR)/include test3f.f
```

##### Linking

```
>ifort -openmp -o test3 test3.o -llis
```

#### MPI

##### Compiling

```
>ifort -c -fpp -DUSE_MPI -I$(INSTALLDIR)/include test3f.f
```

##### Linking

```
>ifort -o test3 test3.o -llis -lmpi
```

#### OpenMP + MPI

##### Compiling

```
>ifort -c -fpp -openmp -DUSE_MPI -I$(INSTALLDIR)/include test3f.f
```

##### Linking

```
>ifort -openmp -o test3 test3.o -llis -lmpi
```

### 3.8 Running

Run the test programs `test3.c` and `test3f.f` in the directory `lis-($VERSION)/test` by entering as follows:

#### Sequential

```
>./test3 -i bicgstab
```

#### OpenMP

```
>env OMP_NUM_THREADS=2 ./test3 -i bicgstab
```

#### MPI

```
>mpirun -np 2 ./test3 -i bicgstab
```

#### OpenMP + MPI

```
>mpirun -np 2 env OMP_NUM_THREADS=2 ./test3 -i bicgstab
```

Then, the following results will be returned:

```
SOLVER : BiCGSTAB
```

```
PRECON : None
```

```
lis_solve is normal end
```

```
iter = 6
```

```
0 1.000000e+000
1 1.000000e+000
2 1.000000e+000
3 1.000000e+000
4 1.000000e+000
5 1.000000e+000
6 1.000000e+000
7 1.000000e+000
8 1.000000e+000
9 1.000000e+000
10 1.000000e+000
11 1.000000e+000
```

## 4 Quadruple Precision Operations

Double precision operations sometimes require a large number of iterations for convergence because of the rounding error. High precision operations are effective for the improvement of convergence. In Lis, we implement quadruple precision operations, which have "double-double" precision[15, 16] by combining two double precision floating point numbers. To use the quadruple precision operations with the same interface as the double precision operations, the coefficient matrix  $A$ , the solution  $x$ , and the right hand side vector  $b$  are assumed to be double precision. Lis implements performance acceleration with SIMD instructions, such as Intel's SSE2[24].

### 4.1 Quadruple Precision Operations

The test program `test4.c` in the directory `lis-($VERSION)/test` supports SSE2 instructions:

#### Double Precision

By entering `>./test4 200 2.0 -precision double`  
the following results will be returned:

```
n=200 gamma=2.000000
Initial vector x = 0
PRECISION : DOUBLE
SOLVER     : BiCG 2
PRECON     : None
STORAGE    : CRS
lis_solve is LIS_MAXITER(code=4)
BiCG: iter      = 1001 iter_double = 1001 iter_quad = 0
BiCG: times     = 2.044368e-02
BiCG: p_times   = 4.768372e-06 (p_c = 4.768372e-06 p_i = 0.000000e+00 )
BiCG: i_times   = 2.043891e-02
BiCG: Residual  = 8.917591e+01
```

#### Quadruple Precision

By entering `>./test4 200 2.0 -precision quad`  
the following results will be returned:

```
n=200 gamma=2.000000
Initial vector x = 0
PRECISION : QUAD
SOLVER     : BiCG 2
PRECON     : None
STORAGE    : CRS
lis_solve is normal end
BiCG: iter      = 230 iter_double = 0 iter_quad = 230
BiCG: times     = 2.267408e-02
BiCG: p_times   = 4.549026e-04 (p_c = 5.006790e-06 p_i = 4.498959e-04 )
BiCG: i_times   = 2.221918e-02
BiCG: Residual  = 6.499145e-11
```

## 5 Matrix Storage Formats

This section describes the matrix storage formats which can be used for the library. Assume that the matrix row (column) number begins with 0 and that the number of the non zero elements of the matrix  $A$  of  $n \times n = (a_{ij})$  is  $nnz$ .

### 5.1 Compressed Row Storage (CRS)

The CRS format uses three arrays `ptr`, `index` and `value` to store data.

- `value` is a double-precision array with a length of  $nnz$ , which stores the non zero elements of the matrix  $A$  along the row.
- `index` is an integer array with a length of  $nnz$ , which stores the column numbers of the non zero elements stored in the array `value`.
- `ptr` is an integer array with a length of  $n + 1$ , which stores the starting points of the rows of the arrays `value` and `index`.

#### 5.1.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 2 shows how the matrix  $A$  in Figure 2 is stored in the CRS format. The program to create this matrix in the CRS format is as follows:

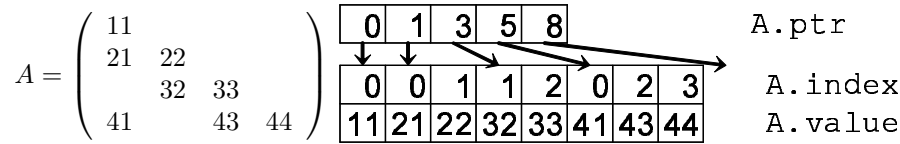


Figure 2: Data structures of CRS.

Sequential and OpenMP

```

1: int      n,nnz;
2: int      *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8;
6: ptr = (int *)malloc( (n+1)*sizeof(int) );
7: index = (int *)malloc( nnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: ptr[0] = 0; ptr[1] = 1; ptr[2] = 3; ptr[3] = 5; ptr[4] = 8;
13: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 1;
14: index[4] = 2; index[5] = 0; index[6] = 2; index[7] = 3;
15: value[0] = 11; value[1] = 21; value[2] = 22; value[3] = 32;
16: value[4] = 33; value[5] = 41; value[6] = 43; value[7] = 44;
17:
18: lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);

```

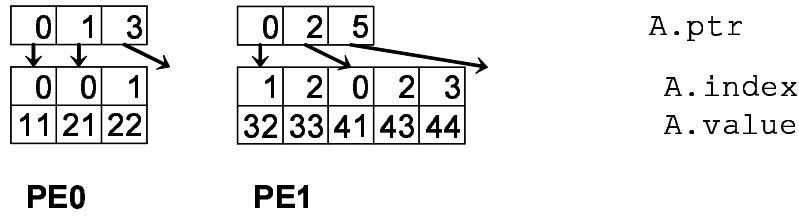


Figure 3: Data structures of CRS.

### 5.1.2 Creating Matrix (MPI)

Figure 3 shows how the matrix  $A$  in Figure 2 is stored in the CRS format between two processors. The program to create this matrix in the CRS format on two processors is as follows:

```

MPI
1: int      i,k,n,nnz,my_rank;
2: int      *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else {n = 2; nnz = 5;}
8: ptr = (int *)malloc( (n+1)*sizeof(int) );
9: index = (int *)malloc( nnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:     ptr[0] = 0; ptr[1] = 1; ptr[2] = 3;
15:     index[0] = 0; index[1] = 0; index[2] = 1;
16:     value[0] = 11; value[1] = 21; value[2] = 22;}
17: else {
18:     ptr[0] = 0; ptr[1] = 2; ptr[2] = 5;
19:     index[0] = 1; index[1] = 2; index[2] = 0; index[3] = 2; index[4] = 3;
20:     value[0] = 32; value[1] = 33; value[2] = 41; value[3] = 43; value[4] = 44;}
21: lis_matrix_set_crs(nnz,ptr,index,value,A);
22: lis_matrix_assemble(A);

```

### 5.1.3 Associating Array

To associate the arrays required by the CRS format with the matrix  $A$ , the following functions are used:

- C            `int lis_matrix_set_crs(int nnz, int row[], int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_crs(integer nnz, integer row(), integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

## 5.2 Compressed Column Storage (CCS)

The CSS format uses three arrays `ptr`, `index` and `value` to store data.

- `value` is a double-precision array with a length of `nnz`, which stores the values for the non zero elements of the matrix  $A$  along the column.
- `index` is an integer array with a length of `nnz`, which stores the row numbers of the non zero elements stored in the array `value`.
- `ptr` is an integer array with a length of  $n + 1$ , which stores the starting points of the rows of the arrays `value` and `index`.

### 5.2.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 4 shows how the matrix  $A$  in Figure 4 is stored in the CCS format. The program to create this matrix in the CCS format is as follows:

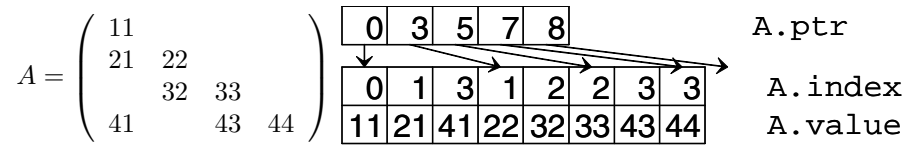


Figure 4: Data structures of CCS.

Sequential and OpenMP

```

1: int      n,nnz;
2: int      *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8;
6: ptr = (int *)malloc( (n+1)*sizeof(int) );
7: index = (int *)malloc( nnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: ptr[0] = 0; ptr[1] = 3; ptr[2] = 5; ptr[3] = 7; ptr[4] = 8;
13: index[0] = 0; index[1] = 1; index[2] = 3; index[3] = 1;
14: index[4] = 2; index[5] = 2; index[6] = 3; index[7] = 3;
15: value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22;
16: value[4] = 32; value[5] = 33; value[6] = 43; value[7] = 44;
17:
18: lis_matrix_set_ccs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);

```



### 5.2.2 Creating Matrix (MPI)

Figure 5 shows how the matrix  $A$  in Figure 4 is stored on two processors. The program to create this matrix in the CCS format between two processors is as follows:

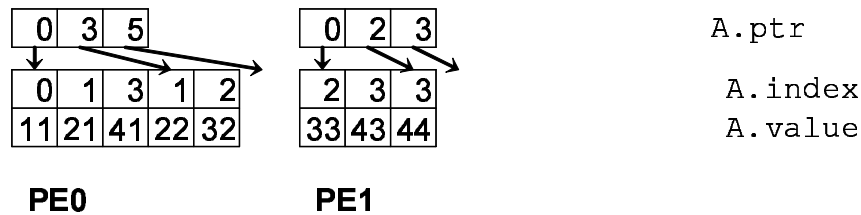


Figure 5: Data structures of CCS.

MPI

```

1: int      i,k,n,nnz,my_rank;
2: int      *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else {n = 2; nnz = 5;}
8: ptr = (int *)malloc( (n+1)*sizeof(int) );
9: index = (int *)malloc( nnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:     ptr[0] = 0; ptr[1] = 3; ptr[2] = 5;
15:     index[0] = 0; index[1] = 1; index[2] = 3; index[3] = 1; index[4] = 2;
16:     value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22; value[4] = 32;
17: } else {
18:     ptr[0] = 0; ptr[1] = 2; ptr[2] = 3;
19:     index[0] = 2; index[1] = 3; index[2] = 3;
20:     value[0] = 33; value[1] = 43; value[2] = 44;
21: lis_matrix_set_ccs(nnz,ptr,index,value,A);
22: lis_matrix_assemble(A);

```

### 5.2.3 Associating Array

To associate the arrays required by the CCS format with the matrix  $A$ , the following functions are used:

- C `int lis_matrix_set_ccs(int nnz, int row[], int index[], LIS_SCALAR value[],`
- Fortran subroutine `lis_matrix_set_ccs(integer nnz, integer row(), integer index(),`  
`LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

### 5.3 Modified Compressed Sparse Row (MSR)

The MSR format is a modified version of the CRS format. The MSR format is different in that it divides the diagonal before storing it. The MSR format uses two arrays `index` and `value` to store data. Assume that `ndz` represents the number of zero elements of the diagonal.

- `value` is a double-precision array with a length of  $nnz + ndz + 1$ , which stores the diagonal of the matrix  $A$  down to the  $n$ -th element. The  $n + 1$ -th element is not used. For the  $n + 2$ -th and after, the values of the non zero elements except the diagonal of the matrix  $A$  are stored along the row.
- `index` is an integer array with a length of  $nnz + ndz + 1$ , which stores the starting points of the rows of the non diagonal elements of the matrix  $A$  down to the  $n + 1$ -th element. For the  $n + 2$ -th and after, it stores the row numbers of the non diagonal elements of the matrix  $A$  stored in the array `value`.

#### 5.3.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 6 shows how matrix  $A$  is stored in the MSR format. The program to create this matrix in the MSR format is as follows:

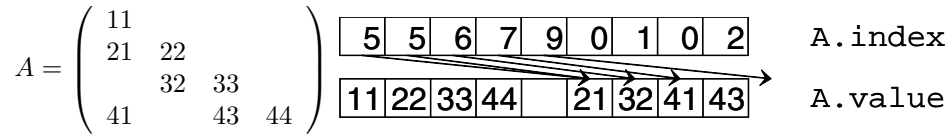


Figure 6: Data structures of MSR.

Sequential and OpenMP

```

1: int      n, nnz, ndz;
2: int      *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8; ndz = 0;
6: index = (int *)malloc( (nnz+ndz+1)*sizeof(int) );
7: value = (LIS_SCALAR *)malloc( (nnz+ndz+1)*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0, &A);
9: lis_matrix_set_size(A, 0, n);
10:
11: index[0] = 5; index[1] = 5; index[2] = 6; index[3] = 7;
12: index[4] = 9; index[5] = 0; index[6] = 1; index[7] = 0; index[8] = 2;
13: value[0] = 11; value[1] = 22; value[2] = 33; value[3] = 44;
14: value[4] = 0; value[5] = 21; value[6] = 32; value[7] = 41; value[8] = 43;
15:
16: lis_matrix_set_msr(nnz, ndz, index, value, A);
17: lis_matrix_assemble(A);

```

### 5.3.2 Creating Matrix (MPI)

Figure 7 shows how the matrix  $A$  in Figure 6 is stored in the MSR format between two processors. The program to create this matrix in the MSR format between two processors is as follows:

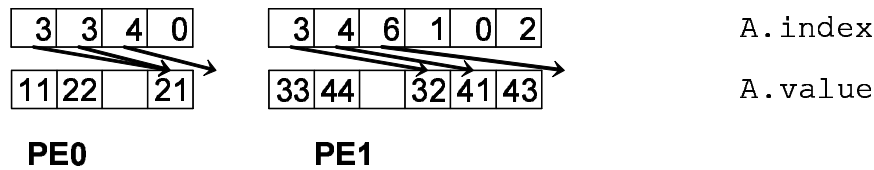


Figure 7: Data structures of MSR.

MPI

```

1: int      i,k,n,nnz,ndz,my_rank;
2: int      *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3; ndz = 0;}
7: else          {n = 2; nnz = 5; ndz = 0;}
8: index = (int *)malloc( (nnz+ndz+1)*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( (nnz+ndz+1)*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:     index[0] = 3; index[1] = 3; index[2] = 4; index[3] = 0;
14:     value[0] = 11; value[1] = 22; value[2] = 0; value[3] = 21;}
15: else {
16:     index[0] = 3; index[1] = 4; index[2] = 6; index[3] = 1;
17:     index[4] = 0; index[5] = 2;
18:     value[0] = 33; value[1] = 44; value[2] = 0; value[3] = 32;
19:     value[4] = 41; value[5] = 43;}
20: lis_matrix_set_msr(nnz,ndz,index,value,A);
21: lis_matrix_assemble(A);

```

### 5.3.3 Associating Array

To associate the arrays required by the MSR format with the matrix  $A$ , the following functions are used:

- C `int lis_matrix_set_msr(int nnz, int ndz, int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_msr(integer nnz, integer ndz, integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

## 5.4 Diagonal (DIA)

DIA uses two arrays **index** and **value** to store data. Assume that *nnd* represents the number of non zero diagonals of the matrix *A*.

- **value** is a double-precision array with a length of  $nnd \times n$ , which stores non zero diagonals of the matrix *A*.
- **index** is an integer array with a length of *nnd*, which stores the offsets from the main diagonal to the other diagonals.

For the OpenMP version, the following modifications have been made: DIA uses two arrays **index** and **value** to store data. Assume that *nprocs* represents the number of threads. *nnd<sub>p</sub>* is the number of non zero diagonals of the partial matrix into which the row block of the matrix *A* is divided. *maxnnd* is the maximum value *nnd<sub>p</sub>*.

- **value** is a double-precision array with a length of  $maxnnd \times n$ , which stores non zero diagonals of the matrix *A*.
- **index** is an integer array with a length of  $nprocs \times maxnnd$ , which stores the offsets from the main diagonal to the other diagonals.

### 5.4.1 Creating Matrix (Sequential)

The right diagram in Figure 8 shows how the matrix *A* in Figure 8 is stored in the DIA format. The program to create this matrix in the DIA format is as follows:

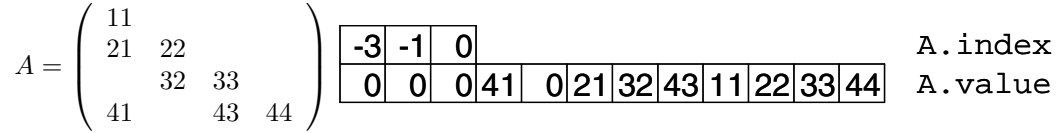


Figure 8: Data structures of DIA.

Sequential

```

1: int      n,nnd;
2: int      *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnd = 3;
6: index = (int *)malloc( nnd*sizeof(int) );
7: value = (LIS_SCALAR *)malloc( n*nnd*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = -3; index[1] = -1; index[2] = 0;
12: value[0] = 0; value[1] = 0; value[2] = 0; value[3] = 41;
13: value[4] = 0; value[5] = 21; value[6] = 32; value[7] = 43;
14: value[8] = 11; value[9] = 22; value[10] = 33; value[11] = 44;
15:
16: lis_matrix_set_dia(nnd,index,value,A);
17: lis_matrix_assemble(A);

```



### 5.4.3 Creating Matrix (MPI)

Figure 10 shows how the matrix  $A$  in Figure 8 is stored in the DIA format between two processors. The program to create this matrix in the DIA format between two processors is as follows:

<table><tr><td>-1</td><td>0</td></tr><tr><td>0</td><td>21</td></tr></table>	-1	0	0	21	<table><tr><td>-3</td><td>-1</td><td>0</td></tr><tr><td>0</td><td>41</td><td>32</td></tr></table>	-3	-1	0	0	41	32	A.index
-1	0											
0	21											
-3	-1	0										
0	41	32										
<table><tr><td>11</td><td>22</td></tr></table>	11	22	<table><tr><td>43</td><td>33</td><td>44</td></tr></table>	43	33	44	A.value					
11	22											
43	33	44										
PE0	PE1											

Figure 10: Data structures of DIA.

MPI

```

1: int          i,n,nnd,my_rank;
2: int          *index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnd = 2;}
7: else          {n = 2; nnd = 3;}
8: index = (int *)malloc( nnd*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( n*nnd*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:     index[0] = -1; index[1] = 0;
14:     value[0] = 0; value[1] = 21; value[2] = 11; value[3] = 22;}
15: else {
16:     index[0] = -3; index[1] = -1; index[2] = 0;
17:     value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 43; value[4] = 33;
18:     value[5] = 44;}
19: lis_matrix_set_dia(nnd,index,value,A);
20: lis_matrix_assemble(A);

```

### 5.4.4 Associating Array

To associate the arrays required by the DIA format with the matrix  $A$ , the following functions are used:

- C `int lis_matrix_set_dia(int nnd, int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_dia(integer nnd, integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

## 5.5 Ellpack-Itpack generalized diagonal (ELL)

ELL uses two arrays `index` and `value` to store data. Assume that `maxnzs` is the maximum value for the number of the non zero elements in the rows of the matrix  $A$ .

- `value` is a double-precision array with a length of  $\text{maxnzs} \times n$ , which stores the non zero elements of the rows of the matrix  $A$  along the column. The first column consists of the number of the first non zero elements of each row. If there is no non zero elements to be stored, then 0 is stored.
- `index` is an integer array with a length of  $\text{maxnzs} \times n$ , which stores the column numbers of the non zero elements stored in the array `value`. If the number of the non zero elements in the  $i$ -th row is  $nnz_i$ , then `index[ $nnz_i \times n + i$ ]` stores row number  $i$ .

### 5.5.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 11 shows how the matrix  $A$  in Figure 11 is stored in the ELL format. The program to create this matrix in the ELL format is as follows:

$$A = \begin{pmatrix} 11 & & & & \\ 21 & 22 & & & \\ & 32 & 33 & & \\ 41 & & 43 & 44 & \end{pmatrix} \quad \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 1 & 0 & 0 & 1 & 2 & 2 & 0 & 1 & 2 & 3 \\ \hline 11 & 21 & 32 & 41 & 0 & 22 & 33 & 43 & 0 & 0 & 0 & 44 \\ \hline \end{array} \quad \begin{array}{l} \text{A.index} \\ \text{A.value} \end{array}$$

Figure 11: Data structures of ELL.

Sequential and OpenMP

```

1: int          n,maxnzs;
2: int          *index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; maxnzs = 3;
6: index = (int *)malloc( n*maxnzs*sizeof(int) );
7: value = (LIS_SCALAR *)malloc( n*maxnzs*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0; index[4] = 0; index[5] = 1;
12: index[6] = 2; index[7] = 2; index[8] = 0; index[9] = 1; index[10] = 2; index[11] = 3;
13: value[0] = 11; value[1] = 21; value[2] = 32; value[3] = 41; value[4] = 0; value[5] = 22;
14: value[6] = 33; value[7] = 43; value[8] = 0; value[9] = 0; value[10] = 0; value[11] = 44;
15:
16: lis_matrix_set_ell(maxnzs,index,value,A);
17: lis_matrix_assemble(A);

```

### 5.5.2 Creating Matrix (MPI)

Figure 12 shows how the matrix  $A$  in Figure 11 is stored in the ELL format. The program to create this matrix in the ELL format between two processors is as follows:

<table><tr><td>0</td><td>0</td><td>0</td><td>1</td></tr><tr><td>11</td><td>21</td><td>0</td><td>22</td></tr></table>	0	0	0	1	11	21	0	22	<table><tr><td>1</td><td>0</td><td>2</td><td>2</td><td>2</td><td>3</td></tr><tr><td>32</td><td>41</td><td>33</td><td>43</td><td>0</td><td>44</td></tr></table>	1	0	2	2	2	3	32	41	33	43	0	44	A.index A.value
0	0	0	1																			
11	21	0	22																			
1	0	2	2	2	3																	
32	41	33	43	0	44																	
PE0	PE1																					

Figure 12: Data structures of ELL.

MPI

```

1: int      i,n,maxnzs,my_rank;
2: int      *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; maxnzs = 2;}
7: else          {n = 2; maxnzs = 3;}
8: index = (int *)malloc( n*maxnzs*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( n*maxnzs*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:     index[0] = 0; index[1] = 0; index[2] = 0; index[3] = 1;
14:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;}
15: else {
16:     index[0] = 1; index[1] = 0; index[2] = 2; index[3] = 2; index[4] = 2;
17:     index[5] = 3;
18:     value[0] = 32; value[1] = 41; value[2] = 33; value[3] = 43; value[4] = 0;
19:     value[5] = 44;}
20: lis_matrix_set_ell(maxnzs,index,value,A);
21: lis_matrix_assemble(A);

```

### 5.5.3 Associating Array

To associate an array required for the ELL format with the matrix  $A$ , the following functions are used:

- C `int lis_matrix_set_ell(int maxnzs, int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_ell(integer maxnzs, integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`



## 5.6 Jagged Diagonal (JDS)

JDS first sorts the non zero elements of the rows in decreasing order of size, and then stores them along the column. JDS uses four arrays **perm**, **ptr**, **index** and **value** to store data. Assume that  $maxn_zr$  represents the maximum value of the number of non zero elements of the matrix  $A$ .

- **perm** is an integer array with a length of  $n$ , which stores the sorted row numbers.
- **value** is a double-precision array with a length of  $nnz$ , which stores the jagged diagonals of the sorted matrix  $A$ . The first jagged diagonal consists of the first non zero elements of each row. The next jagged diagonal consists of the second non zero elements, and so on.
- **index** is an integer array with a length of  $nnz$ , which stores the row numbers of the non zero elements stored in the array **value**.
- **ptr** is an integer array with a length of  $maxn_zr + 1$ , which stores the starting points of the jagged diagonals.

For the OpenMP version, the following modifications have been made: JDS uses four arrays **perm**, **ptr**, **index** and **value** to store data. Assume that  $nprocs$  is the number of threads.  $maxn_zr_p$  is the number of non zero diagonals of the partial matrix into which the row block of the matrix  $A$  is divided.  $maxmaxn_zr$  is the maximum value of  $maxn_zr_p$ .

- **perm** is an integer array with a length of  $n$ , which stores the sorted row numbers.
- **value** is a double-precision array with a length of  $nnz$ , which stores jagged diagonals of the sorted matrix  $A$ . The first jagged diagonal consists of the first non zero elements of each row. The next second jagged diagonal consist of the second non zero elements of each row, and so on.
- **index** is an integer array with a length of  $nnz$ , which stores the row numbers of the non zero elements stored in the array **value**.
- **ptr** is an integer array with a length of  $nprocs \times (maxmaxn_zr + 1)$ , which stores the starting points of the jagged diagonals.

### 5.6.1 Creating Matrix (Sequential)

The right diagram in Figure 13 shows how the matrix  $A$  in Figure 13 is stored in the JDS format. The program to create this matrix in the JDS format is as follows:

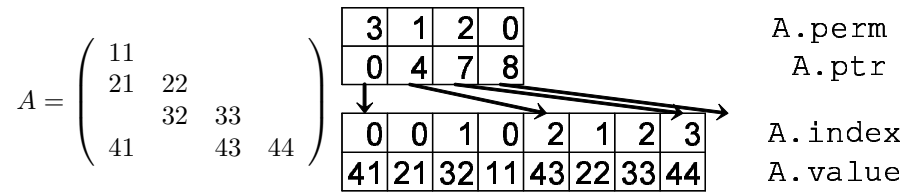


Figure 13: Data structures of JDS.

Sequential

```

1: int      n,nnz,maxnzs;
2: int      *perm,*ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8; maxnzs = 3;
6: perm = (int *)malloc( n*sizeof(int) );
7: ptr = (int *)malloc( (maxnzs+1)*sizeof(int) );
8: index = (int *)malloc( nnz*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
10: lis_matrix_create(0,&A);
11: lis_matrix_set_size(A,0,n);
12:
13: perm[0] = 3; perm[1] = 1; perm[2] = 2; perm[3] = 0;
14: ptr[0] = 0; ptr[1] = 4; ptr[2] = 7; ptr[3] = 8;
15: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0;
16: index[4] = 2; index[5] = 1; index[6] = 2; index[7] = 3;
17: value[0] = 41; value[1] = 21; value[2] = 32; value[3] = 11;
18: value[4] = 43; value[5] = 22; value[6] = 33; value[7] = 44;
19:
20: lis_matrix_set_jds(nnz,maxnzs,perm,ptr,index,value,A);
21: lis_matrix_assemble(A);

```

### 5.6.2 Creating Matrix (OpenMP)

Figure 14 shows how the matrix  $A$  in Figure 13 is stored in the JDS format between two threads. The program to create this matrix in the JDS format between two threads is as follows:

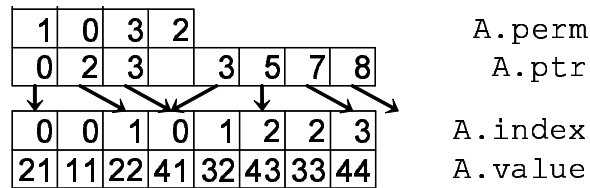


Figure 14: Data structures of JDS.

OpenMP

```

1: int          n, nnz, maxmaxnzs, nprocs;
2: int          *perm, *ptr, *index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; nnz = 8; maxmaxnzs = 3; nprocs = 2;
6: perm = (int *)malloc( n*sizeof(int) );
7: ptr = (int *)malloc( nprocs*(maxmaxnzs+1)*sizeof(int) );
8: index = (int *)malloc( nnz*sizeof(int) );
9: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
10: lis_matrix_create(0, &A);
11: lis_matrix_set_size(A, 0, n);
12:
13: perm[0] = 1; perm[1] = 0; perm[2] = 3; perm[3] = 2;
14: ptr[0] = 0; ptr[1] = 2; ptr[2] = 3; ptr[3] = 0;
15: ptr[4] = 3; ptr[5] = 5; ptr[6] = 7; ptr[7] = 8;
16: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0;
17: index[4] = 1; index[5] = 2; index[6] = 2; index[7] = 3;
18: value[0] = 21; value[1] = 11; value[2] = 22; value[3] = 41;
19: value[4] = 32; value[5] = 43; value[6] = 33; value[7] = 44;
20:
21: lis_matrix_set_jds(nnz, maxmaxnzs, perm, ptr, index, value, A);
22: lis_matrix_assemble(A);

```

### 5.6.3 Creating Matrix (MPI)

Figure 15 shows how the matrix  $A$  in Figure 13 is stored in the JDS format between two processors. The program to create this matrix in the JDS format between two processors is as follows:

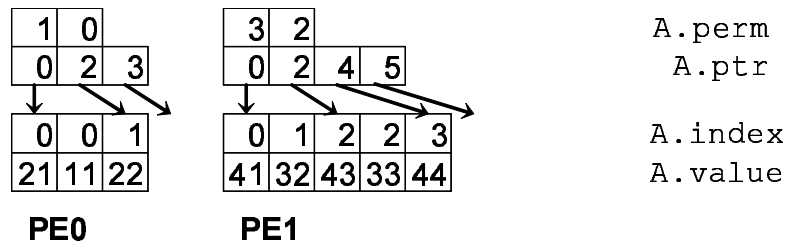


Figure 15: Data structures of JDS.

MPI

```

1: int          i,n,nnz,maxnzs,my_rank;
2: int          *perm,*ptr,*index;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3; maxnzs = 2;}
7: else         {n = 2; nnz = 5; maxnzs = 3;}
8: perm = (int *)malloc( n*sizeof(int) );
9: ptr = (int *)malloc( (maxnzs+1)*sizeof(int) );
10: index = (int *)malloc( nnz*sizeof(int) );
11: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
12: lis_matrix_create(MPI_COMM_WORLD,&A);
13: lis_matrix_set_size(A,n,0);
14: if( my_rank==0 ) {
15:     perm[0] = 1; perm[1] = 0;
16:     ptr[0] = 0; ptr[1] = 2; ptr[2] = 3;
17:     index[0] = 0; index[1] = 0; index[2] = 1;
18:     value[0] = 21; value[1] = 11; value[2] = 22;}
19: else {
20:     perm[0] = 3; perm[1] = 2;
21:     ptr[0] = 0; ptr[1] = 2; ptr[2] = 4; ptr[3] = 5;
22:     index[0] = 0; index[1] = 1; index[2] = 2; index[3] = 2; index[4] = 3;
23:     value[0] = 41; value[1] = 32; value[2] = 43; value[3] = 33; value[4] = 44;}
24: lis_matrix_set_jds(nnz,maxnzs,perm,ptr,index,value,A);
25: lis_matrix_assemble(A);

```

### 5.6.4 Associating Array

To associate an array required for the JDS format with the matrix  $A$ , the following functions are used:

- C `int lis_matrix_set_jds(int nnz, int maxnzs, int perm[], int ptr[], int index[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_jds(integer nnz, integer maxnzs, integer ptr(), integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

## 5.7 Block Sparse Row (BSR)

BSR breaks down the matrix  $A$  into partial matrixes called blocks, with a size of  $r \times c$ . BSR stores non zero blocks, in which at least one non zero element exists, with the same step as that for the CRS format. Assume that  $nr = n/r$  and  $bnnz$  are the numbers of non zero blocks of  $A$ . BSR uses three arrays **bp**tr, **bin**dex and **val**ue to store matrices.

- **value** is a double-precision array with a length of  $bnnz \times r \times c$ , which stores all the elements of the non zero blocks.
- **bin**dex is an integer array with a length of  $bnnz$ , which stores block column numbers of the non zero blocks.
- **bp**tr is an integer array with a length of  $nr + 1$ , which stores the starting points of the block rows in the array **bin**dex.

### 5.7.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 16 shows how the matrix  $A$  in Figure 16 is stored in the BSR format. The program to create this matrix in the BSR format is as follows:

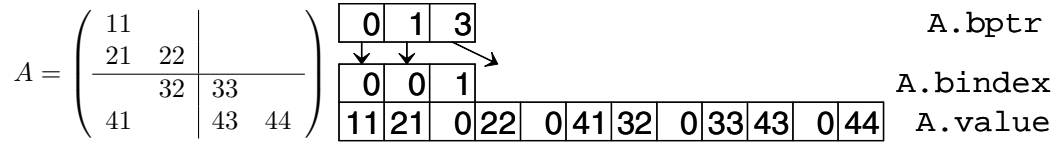


Figure 16: Data structures of BSR.

#### Sequential and OpenMP

```

1: int          n, bnr, bnc, nr, nc, bnnz;
2: int          *bpctr, *bindex;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; bnr = 2; bnc = 2; bnnz = 3; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;
6: bpctr = (int *)malloc( (nr+1)*sizeof(int) );
7: bindex = (int *)malloc( bnnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0, &A);
10: lis_matrix_set_size(A, 0, n);
11:
12: bpctr[0] = 0; bpctr[1] = 1; bpctr[2] = 3;
13: bindex[0] = 0; bindex[1] = 0; bindex[2] = 1;
14: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
15: value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
16: value[8] = 33; value[9] = 43; value[10] = 0; value[11] = 44;
17:
18: lis_matrix_set_bsr(bnr, bnc, bnnz, bpctr, bindex, value, A);
19: lis_matrix_assemble(A);

```

### 5.7.2 Creating Matrix (MPI)

Figure 17 shows how the matrix  $A$  in Figure 16 is stored in the BSR format between two processors. The program to create this matrix in the BSR format between two processors is as follows:

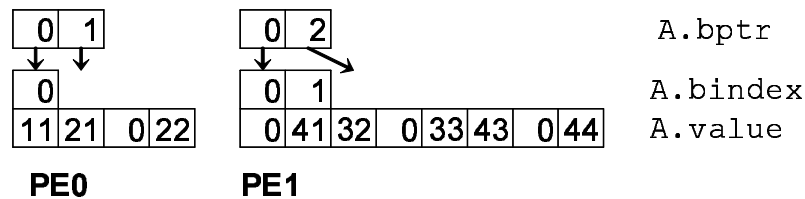


Figure 17: Data structures of BSR.

MPI

```

1: int      n, bnr, bnc, nr, nc, bnnz, my_rank;
2: int      *bptr, *bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
6: if( my_rank==0 ) {n = 2; bnr = 2; bnc = 2; bnnz = 1; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
7: else      {n = 2; bnr = 2; bnc = 2; bnnz = 2; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
8: bptr = (int *)malloc( (nr+1)*sizeof(int) );
9: bindex = (int *)malloc( bnnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD, &A);
12: lis_matrix_set_size(A, n, 0);
13: if( my_rank==0 ) {
14:     bptr[0] = 0; bptr[1] = 1;
15:     bindex[0] = 0;
16:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;}
17: else {
18:     bptr[0] = 0; bptr[1] = 2;
19:     bindex[0] = 0; bindex[1] = 1;
20:     value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 0;
21:     value[4] = 33; value[5] = 43; value[6] = 0; value[7] = 44;}
22: lis_matrix_set_bsr(bnr, bnc, bnnz, bptr, bindex, value, A);
23: lis_matrix_assemble(A);

```

### 5.7.3 Associating Array

To associate the arrays required by the BSR format with the matrix  $A$ , the following functions are used:

- C `int lis_matrix_set_bsr(int bnr, int bnc, int bnnz, int bptr[], int bindex[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_bsr(integer bnr, integer bnc, integer bnnz, integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

## 5.8 Block Sparse Column (BSC)

BSC breaks down the matrix  $A$  into partial matrixes called blocks, with a size of  $r \times c$ . BSC stores non zero blocks, in which at least one non zero block exists, in the same step as that for the CCS format. Assume that  $nc = n/c$  and  $nnzb$  are the numbers of non zero blocks of  $A$ . BSC uses three arrays **bp**tr, **bin**dex and **val**ue to store matrices.

- **value** is a double-precision array with a length of  $nnzb \times r \times c$ , which stores all the elements of the non zero blocks.
- **bin**dex is an integer array with a length of  $nnzb$ , which stores the block row numbers of the non zero blocks.
- **bp**tr is an integer array with a length of  $nc+1$ , which stores the starting points of the block columns in the array **bin**dex.

### 5.8.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 18 shows how the matrix  $A$  in Figure 18 is stored in the BSC format. The program to create this matrix in the BSC format is as follows:

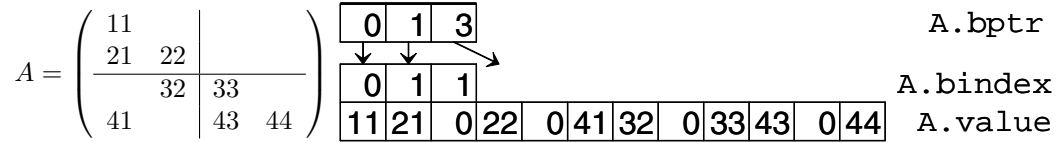


Figure 18: Data structures of BSC.

#### Sequential and OpenMP

```

1: int          n, bnr, bnc, nr, nc, bnnz;
2: int          *bpctr, *bindex;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; bnr = 2; bnc = 2; bnnz = 3; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;
6: bpctr = (int *)malloc( (nc+1)*sizeof(int) );
7: bindex = (int *)malloc( bnnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0, &A);
10: lis_matrix_set_size(A, 0, n);
11:
12: bpctr[0] = 0; bpctr[1] = 1; bpctr[2] = 3;
13: bindex[0] = 0; bindex[1] = 1; bindex[2] = 1;
14: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
15: value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
16: value[8] = 33; value[9] = 43; value[10] = 0; value[11] = 44;
17:
18: lis_matrix_set_bsc(bnr, bnc, bnnz, bpctr, bindex, value, A);
19: lis_matrix_assemble(A);

```

### 5.8.2 Creating Matrix (MPI)

Figure 19 shows how the matrix  $A$  in Figure 18 is stored in the BSC format between two processors. The program to create this matrix in the BSC format between two processors is as follows:

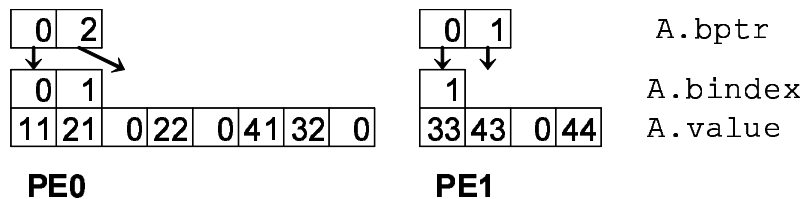


Figure 19: Data structures of BSC.

MPI

```

1: int      n, bnr, bnc, nr, nc, bnnz, my_rank;
2: int      *bptr, *bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
6: if( my_rank==0 ) {n = 2; bnr = 2; bnc = 2; bnnz = 2; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
7: else      {n = 2; bnr = 2; bnc = 2; bnnz = 1; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
8: bptr = (int *)malloc( (nr+1)*sizeof(int) );
9: bindex = (int *)malloc( bnnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD, &A);
12: lis_matrix_set_size(A, n, 0);
13: if( my_rank==0 ) {
14:     bptr[0] = 0; bptr[1] = 2;
15:     bindex[0] = 0; bindex[1] = 1;
16:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
17:     value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;}
18: else {
19:     bptr[0] = 0; bptr[1] = 1;
20:     bindex[0] = 1;
21:     value[0] = 33; value[1] = 43; value[2] = 0; value[3] = 44;}
22: lis_matrix_set_bsc(bnr, bnc, bnnz, bptr, bindex, value, A);
23: lis_matrix_assemble(A);

```

### 5.8.3 Associating Array

To associate the arrays required by the BSC format with the matrix  $A$ , the following functions are used:

- C `int lis_matrix_set_bsc(int bnr, int bnc, int bnnz, int bptr[], int bindex[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_bsc(integer bnr, integer bnc, integer bnnz, integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`



## 5.9 Variable Block Row (VBR)

VBR is the generalized version of BSR. The division points of the rows and columns are given by the arrays `row` and `col`. VBR stores the non zero blocks (the blocks in which at least one non zero block exists) in the same step as that for the CRS format. Assume that  $nr$  and  $nc$  are the numbers of row and column divisions, respectively, and that  $nnzb$  denotes the number of the non zero blocks of  $A$ , and  $nnz$  denotes the total number of the elements of the non zero blocks. VBR uses six arrays `bptr`, `bindex`, `row`, `col`, `ptr` and `value` to store matrices.

- `row` is an integer array with a length of  $nr + 1$ , which stores the starting row number of the block rows.
- `col` is an integer array with a length of  $nc + 1$ , which stores the starting column number of the block columns.
- `bindex` is an integer array with a length of  $nnzb$ , which stores the block column numbers of the non zero blocks.
- `bptr` is an integer array with a length of  $nr + 1$ , which stores the starting points of the block rows in the array `bindex`.
- `value` is a double-precision array with a length of  $nnz$ , which stores all the elements of the non zero blocks.
- `ptr` is an integer array with a length of  $nnzb + 1$ , which stores the starting points of the non zero blocks in the array `value`.

### 5.9.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 20 shows how the matrix  $A$  in Figure 20 is stored in the VBR format. The program to create this matrix in the VBR format is as follows:

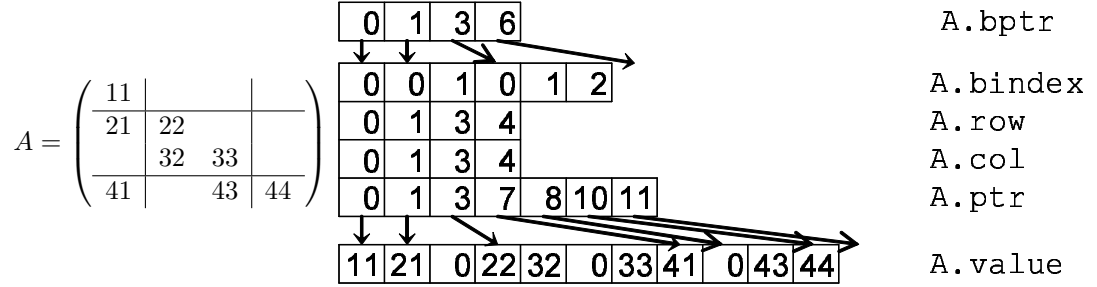


Figure 20: Data structures of VBR.

```

1: int          n,nnz,nr,nc,bnnz;
2: int          *row,*col,*ptr,*bptr,*bindex;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; nnz = 11; bnnz = 6; nr = 3; nc = 3;
6: bptr  = (int *)malloc( (nr+1)*sizeof(int) );
7: row   = (int *)malloc( (nr+1)*sizeof(int) );
8: col   = (int *)malloc( (nc+1)*sizeof(int) );
9: ptr   = (int *)malloc( (bnnz+1)*sizeof(int) );
10: bindex = (int *)malloc( bnnz*sizeof(int) );
11: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
12: lis_matrix_create(0,&A);
13: lis_matrix_set_size(A,0,n);
14:
15: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3; bptr[3] = 6;
16: row[0]  = 0; row[1]  = 1; row[2]  = 3; row[3]  = 4;
17: col[0]  = 0; col[1]  = 1; col[2]  = 3; col[3]  = 4;
18: bindex[0] = 0; bindex[1] = 0; bindex[2] = 1; bindex[3] = 0;
19: bindex[4] = 1; bindex[5] = 2;
20: ptr[0]   = 0; ptr[1]   = 1; ptr[2]   = 3; ptr[3]   = 7;
21: ptr[4]   = 8; ptr[5]   = 10; ptr[6]   = 11;
22: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
23: value[4] = 32; value[5] = 0; value[6] = 33; value[7] = 41;
24: value[8] = 0; value[9] = 43; value[10] = 44;
25:
26: lis_matrix_set_vbr(nnz,nr,nc,bnnz,row,col,ptr,bptr,bindex,value,A);
27: lis_matrix_assemble(A);

```

### 5.9.2 Creating Matrix (MPI)

Figure 21 shows how the matrix  $A$  in Figure 20 is stored in the VBR format between two processors. The program to create this matrix in the VBR format between two processors is as follows:

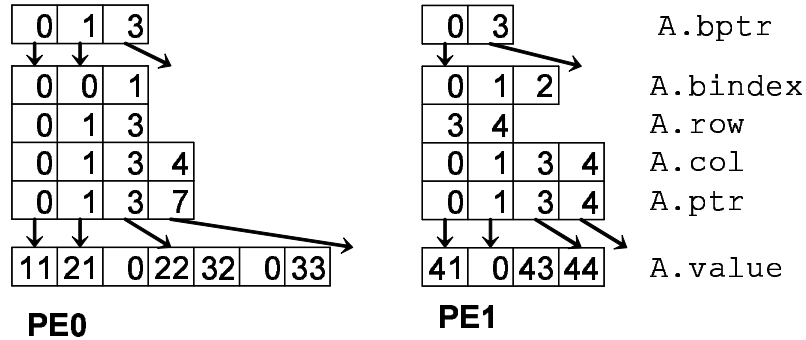


Figure 21: Data structures of VBR.

## MPI

```

1: int          n,nnz,nr,nc,bnnz,my_rank;
2: int          *row,*col,*ptr,*bptr,*bindex;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 7; bnnz = 3; nr = 2; nc = 3;}
7: else          {n = 2; nnz = 4; bnnz = 3; nr = 1; nc = 3;}
8: bptr  = (int *)malloc( (nr+1)*sizeof(int) );
9: row   = (int *)malloc( (nr+1)*sizeof(int) );
10: col   = (int *)malloc( (nc+1)*sizeof(int) );
11: ptr   = (int *)malloc( (bnnz+1)*sizeof(int) );
12: bindex = (int *)malloc( bnnz*sizeof(int) );
13: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
14: lis_matrix_create(MPI_COMM_WORLD,&A);
15: lis_matrix_set_size(A,n,0);
16: if( my_rank==0 ) {
17:     bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
18:     row[0]  = 0; row[1]  = 1; row[2]  = 3;
19:     col[0]  = 0; col[1]  = 1; col[2]  = 3; col[3] = 4;
20:     bindex[0] = 0; bindex[1] = 0; bindex[2] = 1;
21:     ptr[0]   = 0; ptr[1]   = 1; ptr[2]   = 3; ptr[3]   = 7;
22:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
23:     value[4] = 32; value[5] = 0; value[6] = 33;}
24: else {
25:     bptr[0] = 0; bptr[1] = 3;
26:     row[0]  = 3; row[1]  = 4;
27:     col[0]  = 0; col[1]  = 1; col[2]  = 3; col[3] = 4;
28:     bindex[0] = 0; bindex[1] = 1; bindex[2] = 2;
29:     ptr[0]   = 0; ptr[1]   = 1; ptr[2]   = 3; ptr[3]   = 4;
30:     value[0] = 41; value[1] = 0; value[2] = 43; value[3] = 44;}
31: lis_matrix_set_vbr(nnz,nr,nc,bnnz,row,col,ptr,bptr,bindex,value,A);
32: lis_matrix_assemble(A);

```

### 5.9.3 Associating Array

To associate the arrays required by the VBR format with the matrix A, the following functions are used:

- C        `int lis_matrix_set_vbr(int nnz, int nr, int nc, int bnnz, int row[], int col[], int ptr[], int bptr[], int bindex[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_vbr(integer nnz, integer nr, integer nc, integer bnnz, integer row(), integer col(), integer ptr(), integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

## 5.10 Coordinate (COO)

COO uses three arrays `row`, `col` and `value` to store data.

- `value` is a double-precision array with a length of `nnz`, which stores the non zero elements.
- `row` is an integer array with a length of `nnz`, which stores the row numbers of the non zero elements.
- `col` is an integer array with a length of `nnz`, which stores the column numbers of the non zero elements.

### 5.10.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 22 shows how the matrix  $A$  in Figure 22 is stored in the COO format. The program to create this matrix in the COO format is as follows:

$$A = \begin{pmatrix} 11 & & & & & & & & \\ 21 & 22 & & & & & & & \\ & 32 & 33 & & & & & & \\ 41 & & 43 & 44 & & & & & \end{pmatrix} \quad \begin{array}{c} \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 0 & 1 & 3 & 1 & 2 & 2 & 3 & 3 \\ \hline 0 & 0 & 0 & 1 & 1 & 2 & 2 & 3 \\ \hline 11 & 21 & 41 & 22 & 32 & 33 & 43 & 44 \\ \hline \end{array} \\ \begin{array}{l} A.\text{row} \\ A.\text{col} \\ A.\text{value} \end{array} \end{array}$$

Figure 22: Data structures of COO.

Sequential and OpenMP

```

1: int          n,nnz;
2: int          *row,*col;
3: LIS_SCALAR   *value;
4: LIS_MATRIX   A;
5: n = 4; nnz = 8;
6: row = (int *)malloc( nnz*sizeof(int) );
7: col = (int *)malloc( nnz*sizeof(int) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: row[0] = 0; row[1] = 1; row[2] = 3; row[3] = 1;
13: row[4] = 2; row[5] = 2; row[6] = 3; row[7] = 3;
14: col[0] = 0; col[1] = 0; col[2] = 0; col[3] = 1;
15: col[4] = 1; col[5] = 2; col[6] = 2; col[7] = 3;
16: value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22;
17: value[4] = 32; value[5] = 33; value[6] = 43; value[7] = 44;
18:
19: lis_matrix_set_coo(nnz,row,col,value,A);
20: lis_matrix_assemble(A);

```

### 5.10.2 Creating Matrix (MPI)

Figure 23 shows how the matrix  $A$  in Figure 22 is stored in the COO format between two processors. The program to create this matrix in the COO format between two processors is as follows:

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Figure 23: Data structures of COO.

MPI

```

1: int      n,nnz,my_rank;
2: int      *row,*col;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else          {n = 2; nnz = 5;}
8: row  = (int *)malloc( nnz*sizeof(int) );
9: col  = (int *)malloc( nnz*sizeof(int) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:     row[0] = 0; row[1] = 1; row[2] = 1;
15:     col[0] = 0; col[1] = 0; col[2] = 1;
16:     value[0] = 11; value[1] = 21; value[2] = 22;}
17: else {
18:     row[0] = 3; row[1] = 2; row[2] = 2; row[3] = 3; row[4] = 3;
19:     col[0] = 0; col[1] = 1; col[2] = 2; col[3] = 2; col[4] = 3;
20:     value[0] = 41; value[1] = 32; value[2] = 33; value[3] = 43; value[4] = 44;}
21: lis_matrix_set_coo(nnz,row,col,value,A);
22: lis_matrix_assemble(A);

```

### 5.10.3 Associating Array

To associate the arrays required by the COO format with the matrix  $A$ , the following functions are used:

- C `int lis_matrix_set_coo(int nnz, int row[], int col[], LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_coo(integer nnz, integer row(), integer col(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

## 5.11 Dense (DNS)

DNS uses one array `value` to store data.

- `value` is a double-precision array with a length of  $n \times n$ , which stores the elements with priority given to the columns.

### 5.11.1 Creating Matrix (Sequential and OpenMP)

The right diagram in Figure 24 shows how the matrix  $A$  in Figure 24 is stored in the DNS format. The program to create this matrix in the DNS format is as follows:

$$A = \begin{pmatrix} 11 & & & \\ 21 & 22 & & \\ & 32 & 33 & \\ 41 & & 43 & 44 \end{pmatrix} \quad \begin{array}{|c|c|c|c|c|c|c|c|} \hline 11 & 21 & 0 & 41 & 0 & 22 & 32 & 0 \\ \hline 0 & 0 & 33 & 43 & 0 & 0 & 0 & 44 \\ \hline \end{array} \quad \text{A.Value}$$

Figure 24: Data structures of DNS.

Sequential and OpenMP

```

1: int      n;
2: LIS_SCALAR *value;
3: LIS_MATRIX A;
4: n = 4;
5: value = (LIS_SCALAR *)malloc( n*n*sizeof(LIS_SCALAR) );
6: lis_matrix_create(0,&A);
7: lis_matrix_set_size(A,0,n);
8:
9: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 41;
10: value[4] = 0; value[5] = 22; value[6] = 32; value[7] = 0;
11: value[8] = 0; value[9] = 0; value[10] = 33; value[11] = 43;
12: value[12] = 0; value[13] = 0; value[14] = 0; value[15] = 44;
13:
14: lis_matrix_set_dns(value,A);
15: lis_matrix_assemble(A);

```

### 5.11.2 Creating Matrix (MPI)

Figure 25 shows how the matrix  $A$  in Figure 24 is stored in the DNS format between two processors. The program to create this matrix in the DNS format between two processors is as follows:

11	21	0	22	0	41	32	0	A.Value
0	0	0	0	33	43	0	44	
PE0				PE1				

Figure 25: Data structures of DNS.

```

MPI
1: int          n,my_rank;
2: LIS_SCALAR   *value;
3: LIS_MATRIX   A;
4: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
5: if( my_rank==0 ) {n = 2;}
6: else          {n = 2;}
7: value = (LIS_SCALAR *)malloc( n*n*sizeof(LIS_SCALAR) );
8: lis_matrix_create(MPI_COMM_WORLD,&A);
9: lis_matrix_set_size(A,n,0);
10: if( my_rank==0 ) {
11:     value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
12:     value[4] = 0; value[5] = 0; value[6] = 0; value[7] = 0;}
13: else {
14:     value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 0;
15:     value[4] = 33; value[5] = 43; value[6] = 0; value[7] = 44;}
16: lis_matrix_set_dns(value,A);
17: lis_matrix_assemble(A);

```

### 5.11.3 Associating Array

To associate the arrays required by the DNS format with the matrix  $A$ , the following functions are used:

- C            `int lis_matrix_set_dns(LIS_SCALAR value[], LIS_MATRIX A)`
- Fortran subroutine `lis_matrix_set_dns(LIS_SCALAR value(), LIS_MATRIX A, integer ierr)`

## 6 Functions

This section describes the functions which can be employed by users. The return values of the functions in C and the values of `ierr` in Fortran are as follows:

### Return Values

<code>LIS_SUCCESS(0)</code>	Normal termination
<code>LIS_ILL_OPTION(1)</code>	Illegal option
<code>LIS_BREAKDOWN(2)</code>	Breakdown
<code>LIS_OUT_OF_MEMORY(3)</code>	Insufficient working memory
<code>LIS_MAXITER(4)</code>	Did not converge within the maximum number of iterations
<code>LIS_NOT_IMPLEMENTED(5)</code>	Not implemented
<code>LIS_ERR_FILE_IO(6)</code>	File I/O error

### 6.1 Vector Operations

Assume that the size of the vector  $v$  is `global_n` and that the size of the partial vectors divided among `nprocs` processors is `local_n`. `global_n` and `local_n` are called the global size and the local size, respectively.

#### 6.1.1 `lis_vector_create`

<pre>C      int lis_vector_create(LIS_Comm comm, LIS_VECTOR *v) Fortran subroutine lis_vector_create(LIS_Comm comm, LIS_VECTOR v, integer ierr)</pre>
---

#### Description

Create vector

#### Input

<code>LIS_Comm</code>	MPI communicator
-----------------------	------------------

#### Output

<code>v</code>	Vector
<code>ierr</code>	Return code

#### Note

For the sequential and OpenMP versions, the value for `comm` is ignored.



### 6.1.2 lis\_vector\_destroy

```
C      int lis_vector_destroy(LIS_VECTOR v)
Fortran subroutine lis_vector_destroy(LIS_VECTOR v, integer ierr)
```

#### Description

Destroy vector

#### Input

v	Vector to be destroyed
---	------------------------

#### Output

ierr	Return code
------	-------------

### 6.1.3 lis\_vector\_duplicate

```
C      int lis_vector_duplicate(void *vin, LIS_VECTOR *vout)
Fortran subroutine lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR vout,
      integer ierr)
```

#### Description

Create vector which has same information as original

#### Input

vin	Source vector
-----	---------------

#### Output

vout	Destination vector
ierr	Return code

#### Note

The function `lis_vector_duplicate` does not copy the values, but only reserves an area. To copy the values as well, the function `lis_vector_copy` must be used after this function.

#### 6.1.4 lis\_vector\_set\_size

```
C      int lis_vector_set_size(LIS_VECTOR v, int local_n, int global_n)
Fortran subroutine lis_vector_set_size(LIS_VECTOR v, integer local_n,
                                     integer global_n, integer ierr)
```

##### Description

Assign size of vector

##### Input

v	Vector
local_n	Size of partial vector
global_n	Size of global vector

##### Output

ierr	Return code
------	-------------

##### Note

Either `local_n` or `global_n` must be provided. This function can create a vector in one of the following ways: Creates partial vectors of size `local_n` if `local_n` is given, or creates partial vectors divided among a given number of processors, if `global_n` is given.

In the case of the sequential and OpenMP versions, `local_n = global_n`. This means that both `lis_vector_set_size(v,n,0)` and `lis_vector_set_size(v,0,n)` create a vector of size `n`.

#### 6.1.5 lis\_vector\_get\_size

```
C      int lis_vector_get_size(LIS_VECTOR v, int *local_n, int *global_n)
Fortran subroutine lis_vector_get_size(LIS_VECTOR v, integer local_n,
                                     integer global_n, integer ierr)
```

##### Description

Get size of vector

##### Input

v	Vector
---	--------

##### Output

local_n	Size of partial vector
global_n	Size of global vector
ierr	Return code

##### Note

In the case of the sequential and OpenMP versions, `local_n = global_n`.

### 6.1.6 lis\_vector\_get\_range

```
C      int lis_vector_get_range(LIS_VECTOR v, int *is, int *ie)
Fortran subroutine lis_vector_get_range(LIS_VECTOR v, integer is, integer ie,
      integer ierr)
```

#### Description

Get location of partial vector in global vector

#### Input

v	Partial vector
---	----------------

#### Output

is	Location where partial vector $v$ starts in global vector
ie	1+ location where partial vector $v$ ends in global vector
ierr	Return code

#### Note

For the sequential and OpenMP versions, a vector of size  $n$  results in  $is = 0$  and  $ie = n$ .

### 6.1.7 lis\_vector\_set\_value

```
C      int lis_vector_set_value(int flag, int i, LIS_SCALAR value, LIS_VECTOR v)
Fortran subroutine lis_vector_set_value(integer flag, integer i, LIS_SCALAR value,
      LIS_VECTOR v, integer ierr)
```

#### Description

Assign scalar *value* to  $i$ -th row of vector *v*

#### Input

flag	LIS_INS.VALUE : $v[i] = \text{value}$ LIS_ADD.VALUE : $v[i] = v[i] + \text{value}$
i	Location where value is assigned
value	Scalar value to be assigned
v	Destination vector

#### Output

v	Vector with scalar <i>value</i> assigned to $i$ -th row
ierr	Return code

#### Note

For the MPI version, the  $i$ -th row of the global vector must be specified instead of the  $i$ -th row of the partial vector.

### 6.1.8 lis\_vector\_get\_value

```
C      int lis_vector_get_value(LIS_VECTOR v, int i, LIS_SCALAR *value)
Fortran subroutine lis_vector_get_value(LIS_VECTOR v, integer i, LIS_SCALAR value,
      integer ierr)
```

#### Description

Get value of  $i$ -th row of vector  $v$

#### Input

$i$	Location where value should be assigned
$v$	Destination vector

#### Output

value	Value of $i$ -th row
ierr	Return code

#### Note

For the MPI version, the  $i$ -th row of the global vector must be specified rather than the  $i$ -th row of the partial vector.

### 6.1.9 lis\_vector\_set\_values

```
C      int lis_vector_set_values(int flag, int count, int index[],
      LIS_SCALAR value[], LIS_VECTOR v)
Fortran subroutine lis_vector_set_values(integer flag, integer count,
      integer index(), LIS_SCALAR value(), LIS_VECTOR v, integer ierr)
```

#### Description

Assign scalar values  $value[i]$  to the  $index[i]$ -th row of vector  $v$

#### Input

flag	LIS_INS_VALUE : $v[index[i]] = value[i]$ LIS_ADD_VALUE : $v[index[i]] = v[index[i]] + value[i]$
count	Number of elements of array which stores scalar values to be assigned
index	Array which stores location where scalar values should be assigned
value	Array which stores scalar values to be assigned
$v$	Destination vector

#### Output

$v$	Vector with scalar $value[i]$ assigned to its $index[i]$ -th row
ierr	Return code

#### Note

For the MPI version, the  $index[i]$ -th row of the global vector must be specified instead of the  $index[i]$ -th row of the partial vector.

### 6.1.10 lis\_vector\_get\_values

```
C      int lis_vector_get_values(LIS_VECTOR v, int start, int count,  
                                LIS_SCALAR value[])  
Fortran subroutine lis_vector_get_values(LIS_VECTOR v, integer start,  
                                integer count, LIS_SCALAR value(), integer ierr)
```

#### Description

Get scalar values of `start + i`-th row of vector `v`, where  $i = 0, 1, \dots, \text{start} - 1$

#### Input

<code>start</code>	Starting location
<code>count</code>	Number of values to get
<code>v</code>	Destination vector

#### Output

<code>value</code>	Vector to store scalar values
<code>ierr</code>	Return code

#### Note

For the MPI version, the `start + i`-th row of the global vector must be specified rather than the `start + i`-th row of the partial vector.

### 6.1.11 lis\_vector\_scatter

```
C      int lis_vector_scatter(LIS_SCALAR value[], LIS_VECTOR v)
Fortran subroutine lis_vector_scatter(LIS_SCALAR value(), LIS_VECTOR v, integer ierr)
```

#### Description

Assign scalar values of  $i$ -th row of vector  $v$ , where  $i = 0, 1, \dots, \text{global\_n} - 1$

#### Input

value	Array which stores scalar values to be assigned
-------	---

#### Output

v	Destination vector
ierr	Return code

#### Note

### 6.1.12 lis\_vector\_gather

```
C      int lis_vector_gather(LIS_VECTOR v, LIS_SCALAR value[])
Fortran subroutine lis_vector_gather(LIS_VECTOR v, LIS_SCALAR value(), integer ierr)
```

#### Description

Get scalar values of  $i$ -th row of vector  $v$ , where  $i = 0, 1, \dots, \text{global\_n} - 1$

#### Input

v	Source vector
---	---------------

#### Output

value	Vector to store scalar values
ierr	Return code

#### Note

### 6.1.13 lis\_vector\_copy

```
C      int lis_vector_copy(LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_copy(LIS_VECTOR x, LIS_VECTOR y, integer ierr)
```

#### Description

Copy vector:  $y \leftarrow x$

#### Input

x	Source vector
---	---------------

#### Output

y	Destination vector
ierr	Return code

### 6.1.14 lis\_vector\_set\_all

```
C      int lis_vector_set_all(LIS_SCALAR value, LIS_VECTOR x)
Fortran subroutine lis_vector_set_all(LIS_SCALAR value, LIS_VECTOR x, integer ierr)
```

#### Description

Assign scalar value **value** to all elements of vector **x**

#### Input

value	Scalar value to be assigned
v	Target vector

#### Output

v	Vector with <b>value</b> assigned to all elements
ierr	Return code

## 6.2 Matrix Operations

Assume that the size of the matrix  $A$  is  $\text{global\_n} \times \text{global\_n}$  and that the size of each partial matrix divided among  $\text{nprocs}$  processors is  $\text{local\_n} \times \text{global\_n}$ . Here,  $\text{global\_n}$  and  $\text{local\_n}$  are called the number of rows of the global matrix and the number of rows of the partial matrix, respectively.

### 6.2.1 `lis_matrix_create`

```
C      int lis_matrix_create(LIS_Comm comm, LIS_MATRIX *A)
Fortran subroutine lis_matrix_create(LIS_Comm comm, LIS_MATRIX A, integer ierr)
```

#### Description

Create matrix

#### Input

LIS_Comm	MPI communicator
----------	------------------

#### Output

A	Matrix
ierr	Return code

#### Note

For the equential and the OpenMP versions, the value for `comm` is ignored.

### 6.2.2 `lis_matrix_destroy`

```
C      int lis_matrix_destroy(LIS_MATRIX A)
Fortran subroutine lis_matrix_destroy(LIS_MATRIX A, integer ierr)
```

#### Description

Destroy matrix

#### Input

A	Matrix to be destroyed
---	------------------------

#### Output

ierr	Return code
------	-------------



### 6.2.3 lis\_matrix\_duplicate

```
C      int lis_matrix_duplicate(LIS_MATRIX Ain, LIS_MATRIX *Aout)
Fortran subroutine lis_matrix_duplicate(LIS_MATRIX Ain, LIS_MATRIX Aout,
      integer ierr)
```

#### Description

Create matrix which has same information as original

#### Input

Ain	Source matrix
-----	---------------

#### Output

Aout	Destination matrix
ierr	Return code

#### Note

The function `lis_matrix_duplicate` does not copy the values of the elements of the matrix, but only reserves an area. To copy the values of the elements as well, the function `lis_matrix_copy` must be used.

### 6.2.4 lis\_matrix\_malloc

```
C      int lis_matrix_malloc(LIS_MATRIX A, int nnz_row, int nnz[])
Fortran subroutine lis_matrix_malloc(LIS_MATRIX A, integer nnz_row, integer nnz[],
      integer ierr)
```

#### Description

Allocate memory for matrix

#### Input

A	Matrix
nnz_row	Average number of non zero elements
nnz	Array of numbers of non zero elements in each row

#### Output

ierr	Return code
------	-------------

#### Note

Either `nnz_row` or `nnz` must be provided.

### 6.2.5 lis\_matrix\_set\_value

```
C      int lis_matrix_set_value(int flag, int i, int j, LIS_SCALAR value,  
                               LIS_MATRIX A)  
Fortran subroutine lis_matrix_set_value(integer flag, integer i, integer j,  
                                         LIS_SCALAR value, LIS_MATRIX A, integer ierr)
```

#### Description

Assign value to  $(i, j)$  element of matrix

#### Input

flag	LIS_INS.VALUE : $A(i,j) = \text{value}$ LIS_ADD.VALUE : $A(i,j) = A(i,j) + \text{value}$
i	Row number of matrix
j	Column number of matrix
value	Value to be assigned
A	Matrix

#### Output

A	Matrix
ierr	Return code

#### Note

For the MPI version, the  $i$ -th row and the  $j$ -th column of the global matrix must be specified, rather than the  $i$ -th row and the  $j$ -th column of the partial matrix.

The function `lis_matrix_set_value` stores the assigned value in a temporary internal format. For this reason, when `lis_matrix_set_value` is used, the function `lis_matrix_assemble` must be called.

### 6.2.6 lis\_matrix\_assemble

```
C      int lis_matrix_assemble(LIS_MATRIX A)  
Fortran subroutine lis_matrix_assemble(LIS_MATRIX A, integer ierr)
```

#### Description

Build matrix in specified storage format

#### Input

A	Matrix
---	--------

#### Output

A	Matrix built in specified storage format
ierr	Return code

### 6.2.7 lis\_matrix\_set\_size

```
int lis_matrix_set_size(LIS_MATRIX A, int local_n, int global_n)
Fortran subroutine lis_matrix_set_size(LIS_MATRIX A, integer local_n,
integer global_n, integer ierr)
```

#### Description

Assign size of matrix

#### Input

A	Matrix
local_n	Number of rows of partial matrix
global_n	Number of rows of global matrix

#### Output

ierr	Return code
------	-------------

#### Note

Either `local_n` or `global_n` must be provided. This function can create matrices in one of the following two ways: Create partial matrices of size `local_n` x `N` if `local_n` is given, or create partial matrices of size `global_n` x `global_n` is divided among a given number of processors, if `global_n` is given. `N` represents the total sum of `local_n`.

In case of the sequential and OpenMP versions, `local_n = global_n`. This means that both `lis_matrix_set_size(A,n,0)` and `lis_matrix_set_size(A,0,n)` create a matrix of `n` x `n`.

For the MPI version, `lis_matrix_set_size(A,n,0)` creates on the processor  $p$  a partial matrix of size  $n_p \times N$ , where  $N$  is the total sum of  $n_p$ . On the other hand, `lis_matrix_set_size(A,0,n)` creates on the processor  $p$  a partial matrix of size  $m_p \times n$ , where  $m_p$  is the number of the partial matrix, which is determined by the library.

### 6.2.8 lis\_matrix\_get\_size

```
C      int lis_matrix_get_size(LIS_MATRIX A, int *local_n, int *global_n)
Fortran subroutine lis_matrix_get_size(LIS_MATRIX A, integer local_n,
integer global_n, integer ierr)
```

#### Description

Get size of matrix

#### Input

A	Matrix
---	--------

#### Output

local_n	Number of rows of partial matrix
global_n	Number of rows of global matrix
ierr	Return code

#### Note

In case of the sequential and OpenMP versions, `local_n = global_n`.

### 6.2.9 lis\_matrix\_get\_range

```
C      int lis_matrix_get_range(LIS_MATRIX A, int *is, int *ie)
Fortran subroutine lis_matrix_get_range(LIS_MATRIX A, integer is, integer ie,
      integer ierr)
```

#### Description

Get location of partial matrix  $A$  in global matrix

#### Input

A	Partial matrix
---	----------------

#### Output

is	Location where partial matrix $A$ starts in global matrix
ie	1+ location where partial matrix $A$ ends in global matrix
ierr	Return code

#### Note

For the sequential and OpenMP versions, a matrix of  $n \times n$  results in `is = 0` and `ie = n`.

### 6.2.10 lis\_matrix\_set\_type

```
C      int lis_matrix_set_type(LIS_MATRIX A, int matrix_type)
Fortran subroutine lis_matrix_set_type(LIS_MATRIX A, int matrix_type, integer ierr)
```

#### Description

Assign storage format

#### Input

A	Matirx
matrix_type	Storage format

#### Output

ierr	Return code
------	-------------

#### Note

matrix\_type of A is LIS\_MATRIX\_CRS when the matrix is created. The table below shows the available storage formats for matrix\_type.

storage format		matrix_type
Compressed Row Storage	(CRS)	LIS_MATRIX_CRS
Compressed Column Storage	(CCS)	LIS_MATRIX_CCS
Modified Compressed Sparse Row	(MSR)	LIS_MATRIX_MSR
Diagonal	(DIA)	LIS_MATRIX_DIA
Ellpack-Itpack generalized diagonal	(ELL)	LIS_MATRIX_ELL
Jagged Diagonal	(JDS)	LIS_MATRIX_JDS
Block Sparse Row	(BSR)	LIS_MATRIX_BSR
Block Sparse Column	(BSC)	LIS_MATRIX_BSC
Variable Block Row	(VBR)	LIS_MATRIX_VBR
Dense	(DNS)	LIS_MATRIX_DNS
Coordinate	(COO)	LIS_MATRIX_COO

### 6.2.11 lis\_matrix\_get\_type

```
C      int lis_matrix_get_type(LIS_MATRIX A, int *matrix_type)
Fortran subroutine lis_matrix_get_type(LIS_MATRIX A, integer matrix_type,
      integer ierr)
```

#### Description

Get storage format

#### Input

A	Matrix
---	--------

#### Output

matrix_type	Storage format
ierr	Return code

### 6.2.12 lis\_matrix\_set\_blocksize

```
C      int lis_matrix_set_blocksize(LIS_MATRIX A, int bnr, int bnc, int row[],
                                   int col[])
Fortran subroutine lis_matrix_set_blocksize(LIS_MATRIX A, integer bnr, integer bnc,
                                           integer row[], integer col[], integer ierr)
```

#### Description

Assign block size for BSR, BSC, and VBR

#### Input

A	Matrix
bnr	Row block size for BSR (BSC) or number of row blocks for VBR
bnc	Column block size for BSR (BSC) or number of column blocks for VBR
row	Array of row division information about VBR
col	Array of column division information about VBR

#### Output

ierr	Return code
------	-------------

### 6.2.13 lis\_matrix\_convert

```
C      int lis_matrix_convert(LIS_MATRIX Ain, LIS_MATRIX Aout)
Fortran subroutine lis_matrix_convert(LIS_MATRIX Ain, LIS_MATRIX Aout, integer ierr)
```

#### Description

Convert matrix `Ain` into `Aout` of type specified with `lis_matrix_set_type`

#### Input

Ain	Source matrix
-----	---------------

#### Output

Aout	Matrix converted into specified type
ierr	Return code

#### Note

The specification of the converted storage format is set to `Aout` by using `lis_matrix_set_type`. The specification of the block size of BSR, BSC, and VBR is set to `Aout` by using `lis_matrix_set_blocksize`.

In converting the storage type of the source matrix into a specified type, the conversions indicated by one in the table below are performed directly, and the other conversions are made via the indicated types. The conversions with no indication are made via the CRS type.

Src \ Dst	CRS	CCS	MSR	DIA	ELL	JDS	BSR	BSC	VBR	DNS	COO
CRS	1	1	1	1	1	1	1	CCS	1	1	1
COO	1	1	1	CRS	CRS	CRS	CRS	CCS	CRS	CRS	1

#### 6.2.14 lis\_matrix\_copy

```
C      int lis_matrix_copy(LIS_MATRIX Ain, LIS_MATRIX Aout)
Fortran subroutine lis_matrix_copy(LIS_MATRIX Ain, LIS_MATRIX Aout, integer ierr)
```

##### Description

Copy values of matrix elements

##### Input

Ain	Source matrix
-----	---------------

##### Output

Aout	Destination matrix
ierr	Return code

#### 6.2.15 lis\_matrix\_get\_diagonal

```
C      int lis_matrix_get_diagonal(LIS_MATRIX A, LIS_VECTOR d)
Fortran subroutine lis_matrix_get_diagonal(LIS_MATRIX A, LIS_VECTOR d, integer ierr)
```

##### Description

Store diagonal elements of matrix A to vector d

##### Input

A	Matrix
---	--------

##### Output

d	Vector which contains diagonal elements of matrix
ierr	Return code

### 6.2.16 lis\_matrix\_set\_crs

```
C      int lis_matrix_set_crs(int nnz, int ptr[], int index[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_crs(integer nnz, integer row(), integer index(),
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

#### Description

Set matrix A in CRS format

#### Input

nnz	Number of non zero elements
ptr, index, value	Arrays for CRS format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_crs` is used, the function `lis_matrix_assemble` must be called.

### 6.2.17 lis\_matrix\_set\_ccs

```
C      int lis_matrix_set_ccs(int nnz, int ptr[], int index[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_ccs(integer nnz, integer row(), integer index(),
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

#### Description

Set matrix A in CCS format

#### Input

nnz	Number of non zero elements
ptr, index, value	Arrays for CCS format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_ccs` is used, the function `lis_matrix_assemble` must be called.



### 6.2.18 lis\_matrix\_set\_msr

```
C      int lis_matrix_set_msr(int nnz, int ndz, int index[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_msr(integer nnz, integer ndz, integer index(),
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

#### Description

Set matrix A in MSR format

#### Input

nnz	Number of non zero elements
ndz	Number of non zero elements in diagonal
index, value	Arrays for MSR format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_msr` is used, the function `lis_matrix_assemble` must be called.

### 6.2.19 lis\_matrix\_set\_dia

```
C      int lis_matrix_set_dia(int nnd, int index[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_dia(integer nnd, integer index(),
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

#### Description

Set matrix A in DIA format

#### Input

nnd	Number of non zero diagonals
index, value	Arrays for DIA format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_dia` is used, the function `lis_matrix_assemble` must be called.

### 6.2.20 lis\_matrix\_set\_ell

```
C      int lis_matrix_set_ell(int maxnzs, int index[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_ell(integer maxnzs, integer index(),
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

#### Description

Set matrix A in ELL format

#### Input

maxnzs	Maximum number of non zero elements in each row
index, value	Arrays for ELL format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_ell` is used, the function `lis_matrix_assemble` must be called.

### 6.2.21 lis\_matrix\_set\_jds

```
C      int lis_matrix_set_jds(int nnz, int maxnzs, int perm[], int ptr[],
                             int index[], LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_jds(integer nnz, integer maxnzs, integer ptr(),
                                     integer index(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

#### Description

Set matrix A in JDS format

#### Input

nnz	Number of non zero elements
maxnzs	Maximum number of non zero elements in each row
perm, ptr, index, value	Arrays for JDS format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_jds` is used, the function `lis_matrix_assemble` must be called.

### 6.2.22 lis\_matrix\_set\_bsr

```
C      int lis_matrix_set_bsr(int bnr, int bnc, int bnnz, int bptr[], int bindex[],
                             LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_bsr(integer bnr, integer bnc, integer bnnz,
                                     integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A,
                                     integer ierr)
```

#### Description

Set matrix A in BSR format

#### Input

bnr	Row block size
bnc	Column block size
bnnz	Number of non zero blocks
bptr, bindex, value	Arrays for BSR format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_bsr` is used, the function `lis_matrix_assemble` must be called.

### 6.2.23 lis\_matrix\_set\_bsc

```
C      int lis_matrix_set_bsc(int bnr, int bnc, int bnnz, int bptr[], int bindex[],
                             LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_bsc(integer bnr, integer bnc, integer bnnz,
                                     integer bptr(), integer bindex(), LIS_SCALAR value(), LIS_MATRIX A,
                                     integer ierr)
```

#### Description

Set matrix A in BSC format

#### Input

bnr	Row block size
bnc	Column block size
bnnz	Number of non zero blocks
bptr, bindex, value	Arrays for BSC format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_bsc` is used, the function `lis_matrix_assemble` must be called.

#### 6.2.24 lis\_matrix\_set\_vbr

```
C      int lis_matrix_set_vbr(int nnz, int nr, int nc, int bnnz, int row[],
                             int col[], int ptr[], int bptr[], int bindex[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_vbr(integer nnz, integer nr, integer nc,
                                     integer bnnz, integer row(), integer col(), integer ptr(), integer bptr(),
                                     integer bindex(), LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

##### Description

Set matrix A in VBR format

##### Input

nnz	Number of all non zero elements
nr	Number of row blocks
nc	Number of column blocks
bnnz	Number of non zero blocks
row, col, ptr, bptr, bindex, value	Arrays for VBR format
A	Matrix

##### Output

A	Matrix and associated arrays
---	------------------------------

##### Note

When `lis_matrix_set_vbr` is used, the function `lis_matrix_assemble` must be called.

#### 6.2.25 lis\_matrix\_set\_coo

```
C      int lis_matrix_set_coo(int nnz, int row[], int col[], LIS_SCALAR value[],
                             LIS_MATRIX A)
Fortran subroutine lis_matrix_set_coo(integer nnz, integer row(), integer col(),
                                     LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

##### Description

Set matrix A in COO format

##### Input

nnz	Number of non zero elements
row, col, value	Arrays for COO format
A	Matrix

##### Output

A	Matrix and associated arrays
---	------------------------------

##### Note

When `lis_matrix_set_coo` is used, the function `lis_matrix_assemble` must be called.

### 6.2.26 lis\_matrix\_set\_dns

```
C      int lis_matrix_set_dns(LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_dns(LIS_SCALAR value(), LIS_MATRIX A, integer ierr)
```

#### Description

Set matrix A in DNS format

#### Input

value	Array for DNS format
A	Matrix

#### Output

A	Matrix and associated arrays
---	------------------------------

#### Note

When `lis_matrix_set_dns` is used, the function `lis_matrix_assemble` must be called.

## 6.3 Vector and Matrix Operations

### 6.3.1 lis\_vector\_scale

```
C      int lis_vector_scale(LIS_SCALAR alpha, LIS_VECTOR x)
Fortran subroutine lis_vector_scale(LIS_SCALAR alpha, LIS_VECTOR x, integer ierr)
```

#### Description

Multiply vector by **alpha**:  $x \leftarrow \alpha x$

#### Input

alpha	Scalar value
x	Vector to be multiplied by <b>alpha</b>

#### Output

x	Vector multiplied by <b>alpha</b>
ierr	Return code

### 6.3.2 lis\_vector\_dot

```
C      int lis_vector_dot(LIS_VECTOR x, LIS_VECTOR y, LIS_SCALAR *val)
Fortran subroutine lis_vector_dot(LIS_VECTOR x, LIS_VECTOR y, LIS_SCALAR val,
                                integer ierr)
```

#### Description

Calculate inner product:  $val \leftarrow x^T y$

#### Input

x	Vector
y	Vector

#### Output

val	Inner product value
ierr	Return code

### 6.3.3 lis\_vector\_nrm2

```
C      int lis_vector_nrm2(LIS_VECTOR x, LIS_SCALAR *val)
Fortran subroutine lis_vector_nrm2(LIS_VECTOR x, LIS_SCALAR val, integer ierr)
```

#### Description

Calculate 2-norm of vector:  $val \leftarrow \|x\|_2$

#### Input

x	Vector
---	--------

#### Output

val	2-norms of vector
ierr	Return code

#### 6.3.4 lis\_vector\_nrm1

```
C      int lis_vector_nrm1(LIS_VECTOR x, LIS_SCALAR *val)
Fortran subroutine lis_vector_nrm1(LIS_VECTOR x, LIS_SCALAR val, integer ierr)
```

##### Description

Calculate 1-norm of vector:  $val \leftarrow \|x\|_2$

##### Input

x	Vector
---	--------

##### Output

val	1-norm of vector
ierr	Return code

#### 6.3.5 lis\_vector\_axpy

```
C      int lis_vector_axpy(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_axpy(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y,
                                   integer ierr)
```

##### Description

Calculate  $y \leftarrow \alpha x + y$

##### Input

alpha	Scalar value
x, y	Vectors

##### Output

y	$\alpha x + y$ (vector $y$ is overwritten)
ierr	Return code

#### 6.3.6 lis\_vector\_xpay

```
C      int lis_vector_xpay(LIS_VECTOR x, LIS_SCALAR alpha, LIS_VECTOR y)
Fortran subroutine lis_vector_xpay(LIS_VECTOR x, LIS_SCALAR alpha, LIS_VECTOR y,
                                   integer ierr)
```

##### Description

Calculate  $y \leftarrow x + \alpha y$

##### Input

alpha	Scalar value
x, y	Vectors

##### Output

y	$x + \alpha y$ (vector $y$ is overwritten)
ierr	Return code

### 6.3.7 lis\_vector\_axpyz

```
C      int lis_vector_axpyz(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y,
                          LIS_VECTOR z)
Fortran subroutine lis_vector_axpyz(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y,
                                  LIS_VECTOR z, integer ierr)
```

#### Description

Calculate  $z \leftarrow \alpha x + y$

#### Input

alpha	Scalar value
x, y	Vectors

#### Output

z	$x + \alpha y$
ierr	Return code

### 6.3.8 lis\_matrix\_scaling

```
C      int lis_matrix_scaling(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR d, int action)
Fortran subroutine lis_matrix_scaling(LIS_MATRIX A, LIS_VECTOR b,
                                  LIS_VECTOR d, integer action, integer ierr)
```

#### Description

Scale matrix

#### Input

A	Matrix
b	Vector
action	<b>LIS_SCALE_JACOBI</b> : Jacobi scaling $D^{-1}Ax = D^{-1}b$ , where $D$ represents the diagonal of $A = (a_{ij})$ <b>LIS_SCALE_SYMM_DIAG</b> : Diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$ , where $D^{-1/2}$ represents a diagonal matrix with $1/\sqrt{a_{ii}}$ as diagonal

#### Output

d	Vector which contains diagonal elements of $D^{-1}$ or $D^{-1/2}$
ierr	Return code



### 6.3.9 lis\_matvec

```
C          void lis_matvec(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_matvec(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
```

#### Description

Calculate matrix vector product  $y \leftarrow Ax$

#### Input

A	Matrix
x	Vector

#### Output

y	Vector
---	--------

### 6.3.10 lis\_matvect

```
C          void lis_matvect(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_matvect(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
```

#### Description

Calculate transposed matrix vector product  $y \leftarrow A^T x$

#### Input

A	Matrix
x	Vector

#### Output

y	Vector
---	--------

## 6.4 Solving Linear Equation

### 6.4.1 lis\_solver\_create

```
C      int lis_solver_create(LIS_SOLVER *solver)
Fortran subroutine lis_solver_create(LIS_SOLVER solver, integer ierr)
```

#### Description

Create solver

#### Input

None

#### Output

solver	Solver
ierr	Return code

#### Note

`solver` has the information on the solver, the preconditioner, etc.

### 6.4.2 lis\_solver\_destroy

```
C      int lis_solver_destroy(LIS_SOLVER solver)
Fortran subroutine lis_solver_destroy(LIS_SOLVER solver, integer ierr)
```

#### Description

Destroy solver

#### Input

solver	Solver to be destroyed
--------	------------------------

#### Output

ierr	Return code
------	-------------

```
C      int lis_solver_set_option(char *text, LIS_SOLVER solver)
Fortran subroutine lis_solver_set_option(character text, LIS_SOLVER solver,
      integer ierr)
```

## Set options for solver

text	Command line options
------	----------------------

solver	Solver
ierr	Return code

The table below shows the available command line options, where `-i {cg|1}` means `-i cg` or `-i 1` and `-maxiter [1000]` indicates that `-maxiter` defaults to 1,000.

Method	Option	Auxiliary Option	
CG	-i {cg 1}		
BiCG	-i {bicg 2}		
CGS	-i {cgs 3}		
BiCGSTAB	-i {bicgstab 4}		
BiCGSTAB(l)	-i {bicgstabl 5}	-ell [2]	Value for l
GPBiCG	-i {gpbicg 6}		
TFQMR	-i {tfqmr 7}		
Orthomin(m)	-i {orthomin 8}	-restart [40]	Value for restart m
GMRES(m)	-i {gmres 9}	-restart [40]	Value for restart m
Jacobi	-i {jacobi 10}		
Gauss-Seidel	-i {gs 11}		
SOR	-i {sor 12}	-omega [1.9]	Value for relaxation coefficient $\omega$ ( $0 < \omega < 2$ )
BiCGSafe	-i {bicgsafe 13}		
CR	-i {cr 14}		
BiCR	-i {bicr 15}		
CRS	-i {crs 16}		
BiCRSTAB	-i {bicrstab 17}		
GPBiCR	-i {gpbicr 18}		
BiCRSafe	-i {bicrsafe 19}		
FGMRES(m)	-i {fgmres 20}	-restart [40]	Value for restart m
IDR(s)	-i {idrs 21}	-restart [40]	Value for restart s
MINRES	-i {minres 22}		

### Specifying Preconditioner (Default: -p none)

Preconditioner	Option	Auxiliary Option	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	Fill level $k$
SSOR	-p {ssor 3}	-ssor_w [1.0]	Relaxation coefficient $\omega$ ( $0 < \omega < 2$ )
hybrid	-p {hybrid 4}	-hybrid_i [sor]	Linear equation solver
		-hybrid_maxiter [25]	Maximum number of iterations
		-hybrid_tol [1.0e-3]	Convergence criteria
		-hybrid_w [1.5]	Relaxation coefficient $\omega$ for the SOR method ( $0 < \omega < 2$ )
		-hybrid_ell [2]	Value for $l$ of the BiCGSTAB(l) method
		-hybrid_restart [40]	Restart values for GMRES and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	Parameter $\alpha$ for preconditioner of a $I + \alpha S^{(m)}$ type
		-is_m [3]	Parameter $m$ for preconditioner of a $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05]	Drop criteria
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Selection of unsymmetric version (Matrix structure must be symmetric.)
		-saamg_theta [0.05 0.12]	Drop criteria $a_{ij}^2 \leq \theta^2  a_{ii}   a_{jj} $ (symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	Drop criteria
		-iluc_rate [5.0]	Ratio of maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05]	Drop criteria
		-ilut_rate [5.0]	Ratio of maximum fill-in
additive Schwarz	-adds true	-adds_iter [1]	Number of iterations

### Other Options

Option	
-maxiter [1000]	Maximum number of iterations
-tol [1.0e-12]	Convergence criteria
-print [0]	Display of the residual
	-print {none 0} None
	-print {mem 1} Saves the residual history in memory
	-print {out 2} Displays the residual history
	-print {all 3} Saves the residual history and displays it on the screen
-scale [0]	Selection of scaling. The result will overwrite the original matrix and vectors
	-scale {none 0} No scaling
	-scale {jacobi 1} Jacobi scaling $D^{-1}Ax = D^{-1}b$ $D$ represents the diagonal of $A = (a_{ij})$
	-scale {symm_diag 2} Diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$ $D^{-1/2}$ represents a diagonal matrix with $1/\sqrt{a_{ii}}$ as diagonal
-initx_zeros [true]	Behavior of the initial vector $x_0$
	-initx_zeros {false 0} Given values
	-initx_zeros {true 1} All elements are set to 0.
-omp_num_threads [t]	Number of threads $t$ represents the maximum number of threads

**Precision** (Default: `-precision double`)

Precision	Option	Auxiliary Option
DOUBLE	<code>-precision {double 0}</code>	
QUAD	<code>-precision {quad 1}</code>	

#### 6.4.4 lis\_solver\_set\_optionC

```
C      int lis_solver_set_optionC(LIS_SOLVER solver)
Fortran subroutine lis_solver_set_optionC(LIS_SOLVER solver, integer ierr)
```

##### Description

Set options for solver on command line

##### Input

None

##### Output

solver	Solver
ierr	Return code

#### 6.4.5 lis\_solve

```
C      int lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver)
Fortran subroutine lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
                             LIS_SOLVER solver, integer ierr)
```

##### Description

Solve linear equation  $Ax = b$  with specified solver

##### Input

A	Coefficient matrix
b	Right hand side vector
x	Initial vector
solver	Solver

##### Output

x	Solution
solver	Number of iterations, execution time, etc.
ierr	Return code (0)

#### 6.4.6 lis\_solver\_get\_status

```
C      int lis_solver_get_status(LIS_SOLVER solver, int *status)
Fortran subroutine lis_solver_get_status(LIS_SOLVER solver, integer status,
                                         integer ierr)
```

##### Description

Get status from solver

##### Input

solver	Solver
--------	--------

##### Output

status	Number of iterations
ierr	Return code

#### 6.4.7 lis\_solver\_get\_iters

```
C      int lis_solver_get_iters(LIS_SOLVER solver, int *iters)
Fortran subroutine lis_solver_get_iters(LIS_SOLVER solver, integer iters,
                                         integer ierr)
```

##### Description

Get number of iterations from solver

##### Input

solver	Solver
--------	--------

##### Output

iters	Number of iterations
ierr	Return code

#### 6.4.8 lis\_solver\_get\_itersex

```
C      int lis_solver_get_itersex(LIS_SOLVER solver, int *iters, int *iters_double,
                                int *iters_quad)
Fortran subroutine lis_solver_get_itersex(LIS_SOLVER solver, integer iters,
                                         integer iters_double, integer iters_quad, integer ierr)
```

##### Description

Get number of iterations from solver

##### Input

solver	Solver
--------	--------

##### Output

iters	Number of iterations
iters_double	Number of double precision iterations
iters_quad	Number of quadruple precision iterations
ierr	Return code

#### 6.4.9 lis\_solver\_get\_time

```
C      int lis_solver_get_time(LIS_SOLVER solver, double *times, double *itimes,
                              double *ptimes)
Fortran subroutine lis_solver_get_time(LIS_SOLVER solver, real*8 times,
                                       real*8 itimes, real*8 ptimes, integer ierr)
```

##### Description

Get execution time from solver

##### Input

solver	Solver
--------	--------

##### Output

times	Time in seconds for execution
times	Time in seconds for iteration
times	Time in seconds for preconditioning
ierr	Return code



#### 6.4.10 lis\_solver\_get\_timeex

```
C      int lis_solver_get_timeex(LIS_SOLVER solver, double *times, double *itimes,
                                double *ptimes, double *p_c_times, double *p_i_times)
Fortran subroutine lis_solver_get_timeex(LIS_SOLVER solver, real*8 times,
                                         real*8 itimes, real*8 ptimes, real*8 p_c_times, real*8 p_i_times,
                                         integer ierr)
```

##### Description

Get execution time from solver

##### Input

solver	Solver
--------	--------

##### Output

times	Total time in seconds
itimes	Time in seconds for iteration
ptimes	Time in seconds for preconditioning
p_c_times	Time in seconds for creating preconditioner
p_i_times	Time in seconds for iteration in preconditioner
ierr	Return code

#### 6.4.11 lis\_solver\_get\_residualnorm

```
C      int lis_solver_get_residualnorm(LIS_SOLVER solver, LIS_REAL *residual)
Fortran subroutine lis_solver_get_residualnorm(LIS_SOLVER solver, LIS_REAL residual,
                                              integer ierr)
```

##### Description

Get 2-norm of  $b - Ax$  from solver

##### Input

solver	Solver
--------	--------

##### Output

residual	2-norms of $b - Ax$
ierr	Return code

#### 6.4.12 lis\_solver\_get\_rhistory

```
C      int lis_solver_get_rhistory(VECTOR v)
Fortran subroutine lis_solver_get_rhistory(LIS_VECTOR v, integer ierr)
```

##### Description

Store residual norm history of solver

##### Input

None

##### Output

v                                      Vector

ierr                                    Return code

##### Note

The vector **v** must be created in advance with the function **lis\_vector\_create**. When the vector **v** is shorter than the residual history, it stores the residual history in order to the vector **v**.

#### 6.4.13 lis\_solver\_get\_solver

```
C      int lis_solver_get_solver(LIS_SOLVER solver, int *nsol)
Fortran subroutine lis_solver_get_solver(LIS_SOLVER solver, integer nsol,
      integer ierr)
```

##### Description

Get solver number from solver

##### Input

solver                                      Solver

##### Output

nsol                                        Solver number

ierr                                        Return code

##### Note

The number of the solver is as follows:

Solver	Number	Solver	Number
CG	1	Gauss-Seidel	11
BiCG	2	SOR	12
CGS	3	BiCGSafe	13
BiCGSTAB	4	CR	14
BiCGSTAB(l)	5	BiCR	15
GPBiCG	6	CRS	16
TFQMR	7	BiCRSTAB	17
Orthomin(m)	8	GPBiCR	18
GMRES(m)	9	BiCRSafe	19
Jacobi	10	FGMRES(m)	20
IDR(s)	21	MINRES	22

#### 6.4.14 lis\_get\_solvername

```
C      int lis_get_solvername(int nsol, char *name)
Fortran subroutine lis_get_solvername(integer nsol, character name, integer ierr)
```

##### Description

Get solver name from solver number

##### Input

nsol                                        Solver number

##### Output

name                                        Solver name

ierr                                        Return code

## 6.5 Solving Eigenvalue Problem

### 6.5.1 lis\_esolver\_create

```
C      int lis_esolver_create(LIS_ESOLVER *esolver)
Fortran subroutine lis_esolver_create(LIS_ESOLVER esolver, integer ierr)
```

#### Description

Create eigensolver

#### Input

None

#### Output

esolver	Eigensolver
ierr	Return code

#### Note

`esolver` has the information on the eigensolver, the preconditioner, etc.

### 6.5.2 lis\_esolver\_destroy

```
C      int lis_esolver_destroy(LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_destroy(LIS_ESOLVER esolver, integer ierr)
```

#### Description

Destroy eigensolver

#### Input

esolver	Eigensolver to be destroyed
---------	-----------------------------

#### Output

ierr	Return code
------	-------------

### 6.5.3 lis\_esolver\_set\_option

```
C      int lis_esolver_set_option(char *text, LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_set_option(character text, LIS_ESOLVER esolver,
      integer ierr)
```

#### Description

Set options for eigensolver

#### Input

text                                      Command line options

#### Output

esolver                                   Eigensolver

ierr                                      Return code

#### Note

The table below shows the available command line options, where `-e {pi|1}` means `-e pi` or `-e 1` and `-emaxiter [1000]` indicates that `-emaxiter` defaults to 1,000.

**Specifying Eigensolver** (Default: `-e pi`)

Method	Option	Auxiliary Option	
Power Iteration	<code>-e {pi 1}</code>		
Inverse Iteration	<code>-e {ii 2}</code>	<code>-i [bicg]</code>	Linear equation solver
Preconditioned Power Iteration	<code>-e {aii 3}</code>		
Conjugate Gradient	<code>-e {cg 4}</code>		
Lanczos Iteration	<code>-e {li 5}</code>	<code>-ss [10]</code>	Size of subspace
Subspace Iteration	<code>-e {si 6}</code>	<code>-ss [10]</code>	Size of subspace
Conjugate Residual	<code>-e {cr 7}</code>		

### Specifying Preconditioner (Default: -p ilu)

Preconditioner	Option	Auxiliary Option	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	Fill level $k$
SSOR	-p {ssor 3}	-ssor_w [1.0]	Relaxation coefficient $\omega$ ( $0 < \omega < 2$ )
hybrid	-p {hybrid 4}	-hybrid_i [sor]	Linear equation solver
		-hybrid_maxiter [25]	Maximum number of iterations
		-hybrid_tol [1.0e-3]	Convergence criteria
		-hybrid_w [1.5]	Relaxation coefficient $\omega$ for the SOR method ( $0 < \omega < 2$ )
		-hybrid_ell [2]	Value for $l$ of the BiCGSTAB(l) method
		-hybrid_restart [40]	Restart values for GMRES and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	Parameter $\alpha$ for preconditioner of a $I + \alpha S^{(m)}$ type
		-is_m [3]	Parameter $m$ for preconditioner of a $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05]	Drop criteria
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Selection of unsymmetric version (Matrix structure must be symmetric.)
		-saamg_theta [0.05 0.12]	Drop criteria $a_{ij}^2 \leq \theta^2  a_{ii}   a_{jj} $ (symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	Drop criteria
		-iluc_rate [5.0]	Ratio of maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05]	Drop criteria
		-ilut_rate [5.0]	Ratio of maximum fill-in
additive Schwarz	-adds true	-adds_iter [1]	Number of iterations

### Other Options

Option	
-emaxiter [1000]	Maximum number of iterations
-etol [1.0e-5]	Convergence criteria
-eprint [0]	Display of the residual
	-eprint {none 0} None
	-eprint {mem 1} Saves the residual history in memory
	-eprint {out 2} Displays the residual history
	-eprint {all 3} Saves the residual history and displays it on the screen
-ie [ii]	Inner eigensolver used in Lanczos Iteration or Subspace Iteration
	-ie {pi 1} Power Iteration (Subspace Iteration only)
	-ie {ii 2} Inverse Iteration
-shift [0.0]	Amount of shift
	-ie {aii 3} Preconditioned Power Iteration
-initx_ones [true]	Behavior of the initial vector $x_0$
	-initx_ones {false 0} Given values
	-initx_ones {true 1} All elements are set to 1.
-omp_num_threads [t]	Number of threads
	t represents the maximum number of threads

#### 6.5.4 lis\_esolver\_set\_optionC

```
C      int lis_esolver_set_optionC(LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_set_optionC(LIS_ESOLVER esolver, integer ierr)
```

##### Description

Set options for eigensolver on command line

##### Input

None

##### Output

esolver	Eigensolver
ier	Return code

#### 6.5.5 lis\_solve

```
C      int lis_solve(LIS_MATRIX A, LIS_VECTOR x,
                   LIS_REAL eval, LIS_ESOLVER esolver)
Fortran subroutine lis_solve(LIS_MATRIX A, LIS_VECTOR x,
                           LIS_REAL eval, LIS_ESOLVER esolver, integer ierr)
```

##### Description

Solve eigenvalue problem  $Ax = \lambda x$  with specified eigensolver

##### Input

A	Matrix
x	Initial vector
esolver	Eigensolver

##### Output

eval	Extreme eigenvalue
x	Extreme eigenvector
esolver	Number of iterations, execution time, etc.
ier	Return code (0)

### 6.5.6 lis\_esolver\_get\_status

```
C      int lis_esolver_get_status(LIS_ESOLVER esolver, int *status)
Fortran subroutine lis_esolver_get_status(LIS_ESOLVER esolver, integer status,
      integer ierr)
```

#### Description

Get status from eigensolver

#### Input

esolver	Eigensolver
---------	-------------

#### Output

status	Number of iterations
ierr	Return code

### 6.5.7 lis\_esolver\_get\_iters

```
C      int lis_esolver_get_iters(LIS_ESOLVER esolver, int *iters)
Fortran subroutine lis_esolver_get_iters(LIS_ESOLVER esolver, integer iters,
      integer ierr)
```

#### Description

Get number of iterations from eigensolver

#### Input

esolver	Eigensolver
---------	-------------

#### Output

iters	Number of iterations
ierr	Return code



### 6.5.8 lis\_esolver\_get\_itersex

```
C      int lis_esolver_get_itersex(LIS_ESOLVER esolver, int *iters,  
                                int *iters_double, int *iters_quad)  
Fortran subroutine lis_esolver_get_itersex(LIS_ESOLVER esolver, integer iters,  
                                integer iters_double, integer iters_quad, integer ierr)
```

#### Description

Get number of iterations from eigensolver

#### Input

esolver	Eigensolver
---------	-------------

#### Output

iters	Number of iterations
iters_double	Number of double precision iterations
iters_quad	Number of quadruple precision iterations
ierr	Return code

### 6.5.9 lis\_esolver\_get\_time

```
C      int lis_esolver_get_time(LIS_ESOLVER esolver, double *times, double *itimes,  
                              double *ptimes)  
Fortran subroutine lis_esolver_get_time(LIS_ESOLVER esolver, real*8 times,  
                              real*8 itimes, real*8 ptimes, integer ierr)
```

#### Description

Get execution time from eigensolver

#### Input

esolver	Eigensolver
---------	-------------

#### Output

times	Time in seconds for execution
times	Time in seconds for iteration
times	Time in seconds for preconditioning
ierr	Return code

### 6.5.10 lis\_esolver\_get\_timeex

```
C      int lis_esolver_get_timeex(LIS_ESOLVER esolver, double *times,
                                double *itimes, double *ptimes, double *p_c_times, double *p_i_times)
Fortran subroutine lis_esolver_get_timeex(LIS_ESOLVER esolver, real*8 times,
                                real*8 itimes, real*8 ptimes, real*8 p_c_times, real*8 p_i_times,
                                integer ierr)
```

#### Description

Get execution time from eigensolver

#### Input

esolver	Eigensolver
---------	-------------

#### Output

times	Total time in seconds
itimes	Time in seconds for iteration
ptimes	Time in seconds for preconditioning
p_c_times	Time in seconds for creating preconditioner
p_i_times	Time in seconds for iteration in preconditioner
ierr	Return code

### 6.5.11 lis\_esolver\_get\_residualnorm

```
C      int lis_esolver_get_residualnorm(LIS_ESOLVER esolver, LIS_REAL *residual)
Fortran subroutine lis_esolver_get_residualnorm(LIS_ESOLVER esolver,
                                LIS_REAL residual, integer ierr)
```

#### Description

Get 2-norm of  $(\lambda x - Ax)/\lambda$  from eigensolver

#### Input

esolver	Eigensolver
---------	-------------

#### Output

residual	2-norm of $(\lambda x - Ax)/\lambda$
ierr	Return code

### 6.5.12 lis\_esolver\_get\_rhistory

```
C      int lis_esolver_get_rhistory(VECTOR v)
Fortran subroutine lis_esolver_get_rhistory(LIS_VECTOR v, integer ierr)
```

#### Description

Store residual norm history of eigensolver

#### Input

None

#### Output

v	Vector
ierr	Return code

#### Note

The vector **v** must be created in advance with the function **lis\_vector\_create**. When the vector **v** is shorter than the residual history, it stores the residual history in order to the vector **v**.

### 6.5.13 lis\_esolver\_get\_evalues

```
C      int lis_esolver_get_evalues(LIS_ESOLVER esolver, LIS_VECTOR v)
Fortran subroutine lis_esolver_get_evalues(LIS_ESOLVER esolver,
      LIS_VECTOR v, integer ierr)
```

#### Description

Store eigenvalues in vector

#### Input

esolver	Eigensolver
---------	-------------

#### Output

v	Vector which contains eigenvalues
ierr	Return code

#### Note

The vector **v** must be created in advance with the function **lis\_vector\_create**. When the length of the vector **v** is shorter than the number of eigenvalues, it stores the eigenvalues in order to the vector **v**.

### 6.5.14 lis\_esolver\_get\_evectors

```
C      int lis_esolver_get_evectors(LIS_ESOLVER esolver, LIS_VECTOR *v)
Fortran subroutine lis_esolver_get_evectors(LIS_ESOLVER esolver,
      LIS_VECTOR v, integer ierr)
```

#### Description

Storing eigenvectors in array

#### Input

esolver	Eigensolver
---------	-------------

#### Output

v	Array which contains eigenvectors
ierr	Return code

#### Note

Array **\*v** must be created in advance.

### 6.5.15 lis\_esolver\_get\_esolver

```
C      int lis_esolver_get_esolver(LIS_ESOLVER esolver, int *nesol)
Fortran subroutine lis_esolver_get_esolver(LIS_ESOLVER esolver, integer nesol,
      integer ierr)
```

#### Description

Get eigensolver number from eigensolver

#### Input

esolver                                      Eigensolver

#### Output

nesol                                        Eigensolver number

ierr                                        Return code

#### Note

The number of the eigensolver is as follows:

Method	Number
Power Iteration	1
Inverse Iteration	2
Approximate Inverse Iteration	3
Conjugate Gradient	4
Lanczos Iteration	5
Subspace Iteration	6
Conjugate Residual	7

### 6.5.16 lis\_get\_esolvername

```
C      int lis_get_esolvername(int nesol, char *ename)
Fortran subroutine lis_get_esolvername(integer nesol, character ename, integer ierr)
```

#### Description

Get eigensolver name from eigensolver number

#### Input

nesol                                        Eigensolver number

#### Output

name                                        Eigensolver name

ierr                                        Return code

## 6.6 File I/O

### 6.6.1 lis\_input\_matrix

```
C      int lis_input_matrix(LIS_MATRIX A, char *filename)
Fortran subroutine lis_input_matrix(LIS_MATRIX A, LIS_VECTOR x,
      character filename, integer ierr)
```

#### Description

Read matrix from file

#### Input

filename	Name of the source file
----------	-------------------------

#### Output

A	Matrix in specified storage format
x	Solution
ierr	Return code

#### Note

The supported file formats are shown below:

- MatrixMarket format (extended to allow vector data to be read)
- Harwell-Boeing format

### 6.6.2 lis\_input

```
C      int lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, char *filename)
Fortran subroutine lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
      character filename, integer ierr)
```

#### Description

Read matrix and vector from file

#### Input

filename	Name of source file
----------	---------------------

#### Output

A	Matrix in specified storage format
b	Right hand side vector
x	Solution
ierr	Return code

#### Note

The supported file formats are shown below:

- MatrixMarket format (extended to allow vector data to be read)
- Harwell-Boeing format

### 6.6.3 lis\_input\_vector

```
C      int lis_input_vector(LIS_VECTOR v, char *filename)
Fortran subroutine lis_input_vector(LIS_VECTOR v, character filename, integer ierr)
```

#### Description

Read vector from file

#### Input

filename	Name of source file
----------	---------------------

#### Output

v	Vector
ierr	Return code

#### Note

The following formats are supported:

- PLAIN format
- MM format

### 6.6.4 lis\_output

```
C      int lis_output(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, int format,
                    char *filename)
Fortran subroutine lis_output(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
                    integer format, character path, integer ierr)
```

#### Description

Write matrix and vector into file

#### Input

A	Matrix
b	Right hand side vector (If no vector is written into a file, then NULL must be input.)
x	Solution (If no vector is written into a file, then NULL must be input.)
format	File format
	LIS_FMT_MM                      MatrixMarket format
filename	Name of the target file

#### Output

ierr	Return code
------	-------------

### 6.6.5 lis\_output\_vector

```
C      int lis_output_vector(LIS_VECTOR v, int format, char *filename)
Fortran subroutine lis_output_vector(LIS_VECTOR v, integer format,
      character filename, integer ierr)
```

#### Description

Write vector into file

#### Input

v	Vector	
format	File format	
	LIS_FMT_PLAIN	PLAIN format
	LIS_FMT_MM	MM format
	LIS_FMT_LIS	Lis format(ASCII)
filename	Name of target file	

#### Output

ierr	Return code
------	-------------

### 6.6.6 lis\_esolver\_output\_rhistory

```
C      int lis_esolver_output_rhistory(LIS_ESOLVER esolver, char *filename)
Fortran subroutine lis_esolver_output_rhistory(LIS_ESOLVER esolver,
      character filename)
```

#### Description

Write residual history into file

#### Input

esolver	Eigensolver
filename	Name of target file

#### Output

ierr	Return code
------	-------------



## 6.7 Other Functions

### 6.7.1 lis\_initialize

```
C      int lis_initialize(int* argc, char** argv[])
Fortran subroutine lis_initialize(integer ierr)
```

#### Description

Initialize execution environment

#### Input

argc	Number of command line arguments
argv	Command line argument

#### Output

ierr	Return code
------	-------------

### 6.7.2 lis\_finalize

```
C      void lis_finalize()
Fortran subroutine lis_finalize(integer ierr)
```

#### Description

Finalize execution environment

#### Input

None

#### Output

ierr	Return code
------	-------------

### 6.7.3 lis\_wtime

```
C      double lis_wtime()
Fortran function lis_wtime()
```

#### Description

Measure elapsed time

#### Input

None

#### Output

Elapsed time in seconds from given point is returned as double number

#### Note

To measure the processing time, call `lis_wtime` to get the starting time, call it again to get the ending time, and calculate the difference.

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## A File Formats

This section describes the file formats available for the library.

### A.1 MatrixMarket Format

The MatrixMarket format[32] is not designed to store vector data. For this library, it has been extended to handle vector data. Assume that the number of non zero elements for the matrix  $A = (a_{ij})$  of  $M \times N$  is  $L$  and that  $a_{ij} = A(I, J)$ . The file is structured as follows:

```
%%MatrixMarket matrix coordinate real general <-- Header
% <--+
% | Comment lines with 0 or more lines
% <--+
M N L B X <-- Numbers of rows, columns, and
I1 J1 A(I1,J1) <--+ non zero elements (0 or 1) (0 or 1)
I2 J2 A(I2,J2) | Row and column number values
. . . | The index is one origin
IL JL A(IL,JL) <--+
I1 B(I1) <--+
I2 B(I2) | Exists only when B=1
. . . | Row number value
IM B(IM) <--+
I1 X(I1) <--+
I2 X(I2) | Exists only when X=1
. . . | Row number value
IM X(IM) <--+
```

The MatrixMarket file for the matrix  $A$  and the vector  $b$  in Equation (A.1) is structured as follows:

$$A = \begin{pmatrix} 2 & 1 & & \\ 1 & 2 & 1 & \\ & 1 & 2 & 1 \\ & & 1 & 2 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \quad (\text{A.1})$$

```
%%MatrixMarket matrix coordinate real general
4 4 10 1 0
1 2 1.00e+00
1 1 2.00e+00
2 3 1.00e+00
2 1 1.00e+00
2 2 2.00e+00
3 4 1.00e+00
3 2 1.00e+00
3 3 2.00e+00
4 4 2.00e+00
4 3 1.00e+00
1 0.00e+00
2 1.00e+00
3 2.00e+00
4 3.00e+00
```

### A.2 Harwell-Boeing Format

The Harwell-Boeing format inputs and outputs the matrix in the CCS storage format. Assume that the array `value` stores the values of the non zero elements of the matrix  $A$ , the array `index` stores the row indices of the non zero elements and the array `ptr` stores pointers to the beginning of each column in the arrays `value` and `index`. The file is structured as follows:

```

Line 1 (A72,A8)
  1 - 72 Title
  73 - 80 Key
Line 2 (5I14)
  1 - 14 Total number of lines excluding header
  15 - 28 Number of lines for ptr
  29 - 42 Number of lines for index
  43 - 56 Number of lines for value
  57 - 70 Number of lines for right hand side vectors
Line 3 (A3,11X,4I14)
  1 - 3 Matrix type
      Col.1: R Real matrix
            C Complex matrix (Not supported)
            P Pattern only (Not supported)
      Col.2: S Symmetric
            U Unsymmetric
            H Hermitian (Not supported)
            Z Skew symmetric (Not supported)
            R Rectangular (Not supported)
      Col.3: A Assembled
            E Elemental matrices (Not supported)
  4 - 14 Blank space
  15 - 28 Number of rows
  29 - 42 Number of columns
  43 - 56 Number of non zero elements
  57 - 70 0
Line 4 (2A16,2A20)
  1 - 16 Format for ptr
  17 - 32 Format for index
  33 - 52 Format for value
  53 - 72 Format for right hand side vectors
Line 5 (A3,11X,2I14) Only presents if there are right hand side vectors
  1   Right hand side vector type
      F for full storage
      M for same format as matrix (Not supported)
  2   G if a starting vector is supplied
  3   X if an exact solution is supplied
  4 - 14 Blank space
  15 - 28 Number of right hand side vectors
  29 - 42 Number of non zero elements

```

The Harwell-Boeing file for the matrix  $A$  and the vector  $b$  in Equation (A.1) is structured as follows:

```

1-----10-----20-----30-----40-----50-----60-----70-----80
Harwell-Boeing format sample                                     Lis
      8              1              1              4              2
RUA              4              4              10             4
(11i7)          (13i6)          (3e26.18)          (3e26.18)
F              1              0
      1      3      6      9
      1      2      1      2      3      2      3      4      3      4
2.0000000000000000E+00  1.0000000000000000E+00  1.0000000000000000E+00
2.0000000000000000E+00  1.0000000000000000E+00  1.0000000000000000E+00
2.0000000000000000E+00  1.0000000000000000E+00  1.0000000000000000E+00
2.0000000000000000E+00
0.0000000000000000E+00  1.0000000000000000E+00  2.0000000000000000E+00
3.0000000000000000E+00

```

### A.3 xMatrixMarket Format (for Vectors)

The MatrixMarket format[32] has been extended to store vector data. Assume that the vector  $b = (b_i)$  is a vector of order  $N$  and that  $b_i = B(I)$ . The file is structured as follows:

```
%%MatrixMarket vector coordinate real general <-- Header
% <--+
% | Comment lines with 0 or more lines
% <--+
N <-- Number of rows
I1 B(I1) <--+
I2 B(I2) | Row number value
. . . | The index is one origin
IN B(IN) <--+
```

The MatrixMarket file for the vector  $b$  in Equation (A.1) is structured as follows:

```
%%MatrixMarket vector coordinate real general
4
1 0.00e+00
2 1.00e+00
3 2.00e+00
4 3.00e+00
```

### A.4 PLAIN Format (for Vectors)

The PLAIN format is designed to write vector values in order. Assume that the vector  $b = (b_i)$  is a vector of order  $N$  and that  $b_i = B(I)$ . The file is structured as follows:

```
B(1) <--+
B(2) | Vector value
. . . |
B(N) <--+
```

For the vector  $b$  in Equation (A.1), the file is structured as follows:

```
0.00e+00
1.00e+00
2.00e+00
3.00e+00
```